# 國立成功大學

# 114學年度碩士班招生考試試題

編　號：134

系　所：電腦與通信工程研究所

科　目：通訊數學

日　期：0210

節　次：第 1 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (10%) If $X$ has the Poisson distribution with parameter $\lambda = 3$, what is the probability mass function (PMF) of the random variable $Y = |X - 3|$.

2. (10%) Consider $n$ Bernoulli trials with success probability $p$. Find the conditional PMF of the trial on which the first success occurs, given that the number of success is $k$.

3. (15%) Let $X$, $Y$ and $Z$ be the three velocity components of a gas molecule. $X$, $Y$ and $Z$ are independent and identically distributed normal random variables with common expectation zero and variance $\sigma^2$. Determine the probability density function (PDF) of the speed defined as $\sqrt{X^2 + Y^2 + Z^2}$ of a gas molecule.

4. (15%) Two people agree to meet at a specified place at noon. Let's assume that the times that the two people arrive are independent random variable, each having the PDF as follows,

$$f_T(t) = \frac{1}{5}\left(1 - \frac{|t|}{5}\right), \quad -5 < t < 5 \tag{1}$$

where time (i.e. $t$ in (1)) is measured in minutes relative to noon. Determine the probability that the first person to arrive will wait more than 5 minutes before the other person arrives.

5. (30%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) If the determinant of a square matrix $A$ is $\det(A)$, then $\det(-A) = -\det(A)$.

   (b) For two square matrices $A$ and $B$ of the same size, if $AB = O$, the zero matrix, then we also have $BA = O$.

   (c) A vector space has a unique basis.

   (d) If $T$ is a linear operator on a vector space $V$, then $T^2$ is also a linear operator on $V$.

   (e) Let $S$ be a linearly independent set in a vector space $V$. If $\mathbf{u}$ is a vector in $V$ and $\mathbf{u} \notin \text{span}(S)$, then $S \cup \{\mathbf{u}\}$ is also a linearly independent set.

   (f) In an inner product space, we have the associated Schwartz Inequality.

6. (20%) Consider a linear transformation (operator) $T : \mathbb{P}_1 \mapsto \mathbb{P}_1$ by

$$T(a_0 + a_1 t) = a_1 + (-2a_0 + 3a_1)t\,,$$

where $\mathbb{P}_1$ denotes the space of polynomials of degrees less than or equal to one. Find the eigenvalues of $T$ and give an eigenvector (polynomial) for each eigenvalue.