國立成功大學

114學年度碩士班招生考試試題

編　號：135

系　所：電腦與通信工程研究所

科　目：通訊系統

日　期：0210

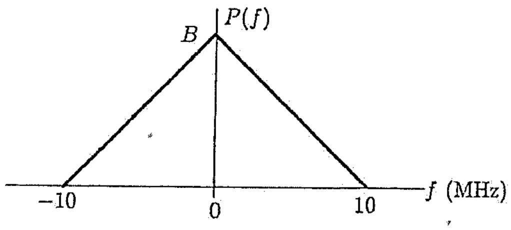
節　次：第 2 節

注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (25%) Answer the following questions.
   (a) Prove the convolution theorem of the continuous-time Fourier transform. [15 points]
   (b) Let us define * as the convolution operator and $\tau > 0$ is a constant. Find $2\tau \text{sinc}(2\tau t) * \tau \text{sinc}(\tau t)$ where $\text{sinc}(x) \triangleq \dfrac{\sin(\pi x)}{\pi x}$. [10 points]

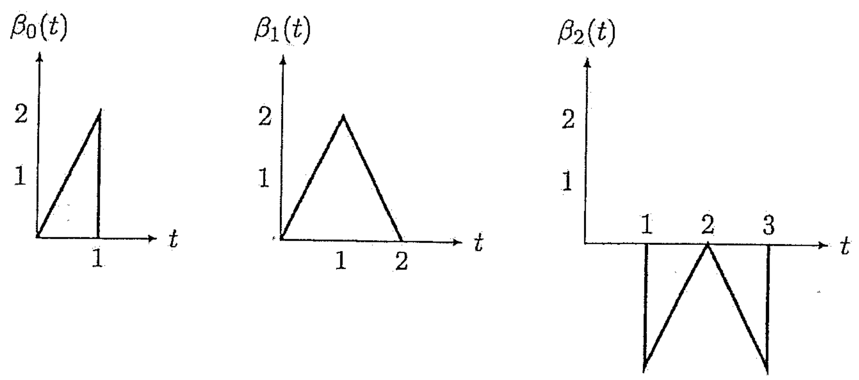
2. (25%) Consider the random process $n(t) = A\cos(2\pi f_o t + \Theta)$ where A and f_0 are constants, and Θ is a random variable that is uniformly distributed in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. Answer the following questions regarding this random process.
   (a) Find the statistical-average auto-correlation function. [10 points]
   (b) Is this random process stationary? Justify your answer. (Hint: Use the answer from (a).) [5 points]
   (c) Find the time-average auto-correlation function. [5 points]
   (d) Is this an ergodic process? Justify your answer. (Hint: Use the answer from (c).) [5 points]

3. (15%) Consider the **overall** pulse $P(f) = G(f)H(f)C(f)$ shown below, where $G(f)$ is the transmit filter, $C(f)$ is the receive filter, and $H(f)$ is the channel.



   (a) Can this pulse be used to communicate with **zero ISI** (Inter-Symbol Interference)? If so, at what baud rate R? [5 points]
   (b) Suppose this pulse is used to construct a transmit filter $g(t)$ and a corresponding receiver matched filter $c(t)$ in an ideal channel, i.e., $H(f) = 1$. Sketch $|G(f)|$. [5 points]
   (c) What is the energy in the pulse $g(t)$ obtained in part (b)? [5 points]

4. (20%) Consider the signal set shown in figure below

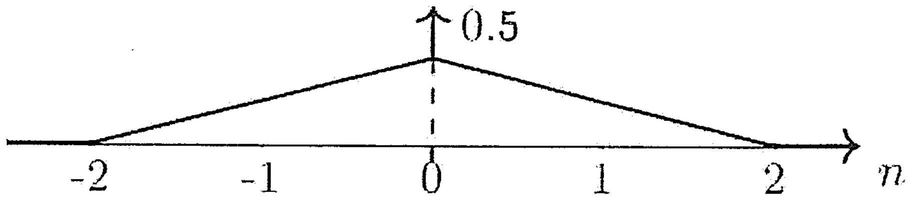


(a) Using the **Gram-Schmidt procedure**, find an orthonormal basis for the space spanned by the waveforms $\beta_0(t), \beta_1(t), \beta_2(t)$ given above. **Please start with** $\beta_0(t)$**, then** $\beta_1(t)$**, and so on.** [10 points]

(b) In your chosen orthonormal basis, let $w_0(t)$ and $w_1(t)$ be represented by the codewords $\mathbf{c}_0 = [3, -1, 1]^\top$ and $\mathbf{c}_1 = [-1, 2, 3]^\top$ respectively. Plot $w_0(t)$ and $w_1(t)$. [5 points]

(c) Compute the (standard) inner products $\langle \mathbf{c}_0, \mathbf{c}_1 \rangle$ and $\langle w_0(t), w_1(t) \rangle$ and compare them. [5 points]

5. (15%) Suppose an M-ary PAM receiver observes $r = a + n$, where $a \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ with uniform a priori probabilities $P(a) = 1/M$, and where the noise is independent of a with the triangular probability density function shown in the figure.



(a) **True or false:** The minimum-distance detector will make the same decisions as the maximum-likelihood (ML) detector. [3 points]

(b) Find the probability (call it $\mathbb{Q}$) that the noise exceeds one, i.e., find $\mathbb{Q} = P(n > 1)$. [3 points]

(c) Find the probability of error, P_e, for the ML detector when $M = 2$. [3 points]

(d) Find the probability of error, P_e, for the ML detector when $M = 4$. [3 points]

(e) Find an expression for the probability of error, P_e, for the ML detector, expressed as a function of M, when $M \in \{2, 4, 8, 16, 32, 64, 128, \ldots\}$ is an unspecified power of two. [3 points]