

國立成功大學

114學年度碩士班招生考試試題

編 號：140

系 所：電腦與通信工程研究所

科 目：機率與線性代數

日 期：0210

節 次：第 3 節

注 意：1.不可使用計算機
2.請於答案卷(卡)作答，於
試題上作答，不予計分。

1. (30%) The joint pdf of RVs X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & \text{if } 0 \leq y \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

- Find $E(X)$ and $\text{Var}(X)$. (5%)
 - Find $E(Y)$ and $\text{Var}(Y)$. (5%)
 - Find $\text{Cov}(X,Y)$. (10%)
 - Find $\text{Var}(X+Y)$. (10%)
2. (20%) A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set $\{A, A-, B+, B, B-, C+\}$, with equal probability and independent of the other papers. At least how many papers do you expect to hand in when you receive each possible grade at least once?

3. (21%, 7% each) Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$.

- Find a matrix P such that $P^{-1}AP$ is diagonal.
 - Let $n \geq 1$ be an arbitrary integer. Find A^n .
 - Use the result of part (b) to determine the matrix $B = \lim_{n \rightarrow \infty} A^n$.
4. (29%) Let A be the matrix given by $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$ with $\text{rank } A = 2$.
- Find an SVD of A . (8%)
 - Compute A^+ . (7%)
 - Find a least-squares solution for $Ax = b$, where $b = [1, 0]^T$. (7%)
 - Find the least-squares error for part (c). (7%)