

# 國立成功大學

## 114學年度碩士班招生考試試題

編 號：130、141

系 所：電機工程學系  
電腦與通信工程研究所

科 目：工程數學

日 期：0210

節 次：第 3 節

注 意：1.不可使用計算機  
2.請於答案卷(卡)作答，於  
試題上作答，不予計分。

1. (20%) Consider the following differential equation and answer the following questions.

$$x^2 y'' - 5xy' + 8y = 0 \quad \forall x \in (-1, 1)$$

- (a)(10%) Find the general solution as the form  $y = c_1 y_1 + c_2 y_2$ , where  $c_1$  and  $c_2$  are constants.

- (b)(10%) Explain why the Wroskian determinant of above solutions (i.e.,  $y_1$  and  $y_2$ ) can be  $W(0) = 0$ .

2. (20%) Solve the following differential equation.

$$(1 - t)y'' + ty' - y = 0; y(0) = 3; y'(0) = -1.$$

3. (20%) Find the eigenvalues and eigenfunction of the following differential equation.

$$(e^{2x} y')' + \lambda e^{2x} y = 0; y(0) = y(\pi) = 0.$$

4. (20%) Solve the following differential equation.

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2} &= 16 \frac{\partial^2 y}{\partial x^2} \text{ for } -\infty < x < \infty, t \geq 0 \\ y(x, 0) &= 8e^{-5|x|} \text{ for } -\infty < x < \infty \\ \frac{\partial y}{\partial t}(x, 0) &= 0. \end{aligned}$$

5. (20%) Find the expression of the solution to the following differential equation.

$$\begin{aligned} \frac{\partial u}{\partial t} &= k \frac{\partial^2 u}{\partial x^2} \text{ for } -\infty < x < \infty, t > 0 \\ u(x, 0) &= f(x) \text{ for } -\infty < x < \infty. \end{aligned}$$

You can use the following tool.

$$\int_{-\infty}^{\infty} e^{-x^2} \cos\left(\frac{\alpha x}{\beta}\right) dx = \sqrt{\pi} e^{-\alpha^2/4\beta^2} \text{ for real } \alpha \text{ and } \beta, \text{ with } \beta \neq 0.$$