國立成功大學

115學年度碩士班招生考試試題

編　號：132

系　所：通訊工程研究所

科　目：通訊數學

日　期：0203

節　次：第 1 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. Consider Bernoulli trials with success probability p. Let X be the number of trials until the first success and Y be the number of trials from the first success until the second success.

   (a) (15%) Prove that for any two positive integers x and y, the events $\{X = x\}$ and $\{Y = y\}$ are independent.

   (b) (15%) Obtain the Probability Mass Function of the random variable $X + Y$.

2. Let X and Y be the x and y coordinates, respectively, of a point selected at random (i.e. uniformly) from the upper half of the unit circle, that is from the set $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1, y > 0\}$

   (a) (10%) Find the joint Probability Density Function of X and Y.

   (b) (10%) Determine the probability that the point obtained lies in the triangle with vertices $(-1, 0)$, $(0, 1)$ and $(1, 0)$.

3. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) For two square matrices A and B of the same size, if AB is convertible, then BA is also invertible.

   (b) If T are S are two linear operators on a vector space V. Then $T^2 + 3S$ is also a linear operator on V.

   (c) Every matrix has its singular value decomposition. (If we do not consider the zero matrix.)

   (d) Every square matrix can be diagonalized.

4. (15%) If A is a positive-definite matrix, show that A is also an invertible matrix.

5. (15%) Consider two sequences $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ in $\mathbb{C}^n$, where $\mathbb{C}$ denotes the set of complex numbers. Choose the true statement(s) in the following. (a) $\left|\sum_{k=1}^{n} a_k b_k\right|^2 \leq \sum_{k=1}^{n} |a_k|^2 \sum_{k=1}^{n} |b_k|^2$ (b) $\left|\sum_{k=1}^{n} a_k \overline{b}_k\right|^2 \leq \sum_{k=1}^{n} |a_k|^2 \sum_{k=1}^{n} |b_k|^2$ (c) $\left|\sum_{k=1}^{n} \overline{a}_k b_k\right|^2 \leq \sum_{k=1}^{n} |a_k|^2 \sum_{k=1}^{n} |b_k|^2$ (d) $\left|\sum_{k=1}^{n} \overline{a}_k \overline{b}_k\right|^2 \leq \sum_{k=1}^{n} |a_k|^2 \sum_{k=1}^{n} |b_k|^2$