

國立成功大學

115學年度碩士班招生考試試題

編 號：133

系 所：通訊工程研究所

科 目：通訊系統

日 期：0203

節 次：第 2 節

注 意：1. 可使用計算機
2. 請於答案卷(卡)作答，於
試題上作答，不予計分。

1. (30%) A message signal $m(t) = 3 \cos(10\pi t)$ is transmitted using amplitude modulation (AM). The transmitted modulated signal is given by $x(t) = (1 + 2am(t)) \cdot \cos\left(10^6 \pi t + \frac{\pi}{2}\right)$ where $a > 0$ is a constant parameter. We assume an ideal channel, where the received signal is identical to the transmitted signal.
- (a) Demonstrate how you can coherently demodulate this AM signal. You can assume perfect carrier synchronization at the receiver. Show important intermediate steps in your derivation. (10%)
- (b) Determine the range of the constant parameter a for which the message signal can be successfully recovered (i.e., demodulated) by the ideal envelope detector. (10%)
- (c) Find the power efficiency of this AM signal. (10%) (Hint: The power efficiency is defined as the percentage of the power of the information-bearing signal in the modulated signal.)
2. (20%) Suppose that $x[n]$ is obtained by uniformly sampling $x(t) = \cos(100\pi t)$ at a sampling frequency of $\frac{900}{7}$ Hz. Answer the following questions.
- (a) Is $x[n]$ periodic? If so, find the fundamental period. If not, justify your answer. (10%)
- (b) Find and plot the discrete-time Fourier transform (DTFT) of $x[n]$. (10%)
3. (15%) Suppose that two *independent* random processes $X(t)$ and $Y(t)$ are input to a multiplier as shown in Fig. 1.

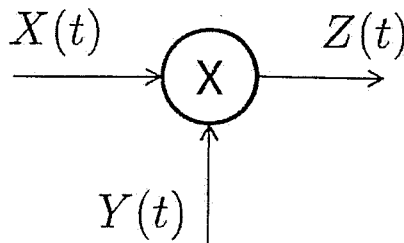


Fig. 1

- (a) Find an expression for the autocorrelation and power spectrum of the output $Z(t)$ in terms of the autocorrelation and power spectrum of $X(t)$ and $Y(t)$. Suppose that $X(t)$ and $Y(t)$ both have the power spectrum $S_X(f) = S_Y(f) = A \text{rect}\left(\frac{f}{2W}\right)$. (5% + 5%)
- (b) The output $Z(t)$ is sampled at intervals of T_s seconds to yield the sample sequence $\{Z_k\}$, where $Z_k = Z(kT_s)$. What sample spacing will ensure that the Z_k are mutually independent random variables? (5%)
4. (15%) An optimum binary digital modulation system is given that is impacted by two types of noise. The first noise type has triangular probability density function between -1 and 1 . The second noise type has triangular probability density function between -2 and 2 . Bits 0 and 1 are transmitted with equal probability. Bit ' 0 ' is mapped to the symbol -1 and bit ' 1 ' is mapped to the symbol 1 . If bit ' 0 ' is transmitted, the received symbol is impacted by the first noise type. If bit ' 1 ' is transmitted, the received

symbol is impacted by the second noise type. The receiver operates by comparing with a threshold.

- (a) What is the probability of error if the threshold is 0? (3%)
- (b) What is the optimum threshold value that the receiver can use to minimize the probability of error? (8%)
- (c) What is the bit error rate at that optimum threshold? In other words, what is the lowest bit error rate that this system can achieve? (4%)

5. (20%) Consider the basis waveforms $\varphi_1(t) = \frac{\sin(\pi t)}{\pi t}$ and $\varphi_2(t) = \varphi_1(t-1)$. Suppose a 4-ary modulation scheme is defined in terms of these waveforms as follows:

$$s_1(t) = A\varphi_1(t)$$

$$s_2(t) = A\varphi_2(t)$$

$$s_3(t) = A(\varphi_1(t) + \varphi_2(t))$$

$$s_4(t) = 0.$$

The factor of A is a scaling constant, common to all signals.

- (a) Verify that $\varphi_1(t)$ and $\varphi_2(t)$ are **orthogonal**. (6%)
- (b) Find the bit rate R_b achieved by this modulation scheme. (2%)
- (c) Find the energy per bit E_b , expressed as a function of the parameter A . (2%)
- (d) Draw the signal constellation for this modulation scheme and assign a Gray coding to the symbols. (2%)
- (e) Find an exact expression, in terms of $Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{\beta^2}{2}} d\beta$, for the **symbol-error probability** P_e of the ML detector when this 4-ary modulation scheme is used over an AWGN channel with noise power spectral density (PSD) $N_0/2$. (4%)
- (f) Find an *exact* expression, in terms of $Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{\beta^2}{2}} d\beta$, for the **bit-error probability** P_b of ML detector when this 4-ary modulation scheme is used with a Gray code over an AWGN channel, expressed as a function of E_b/N_0 . (4%)