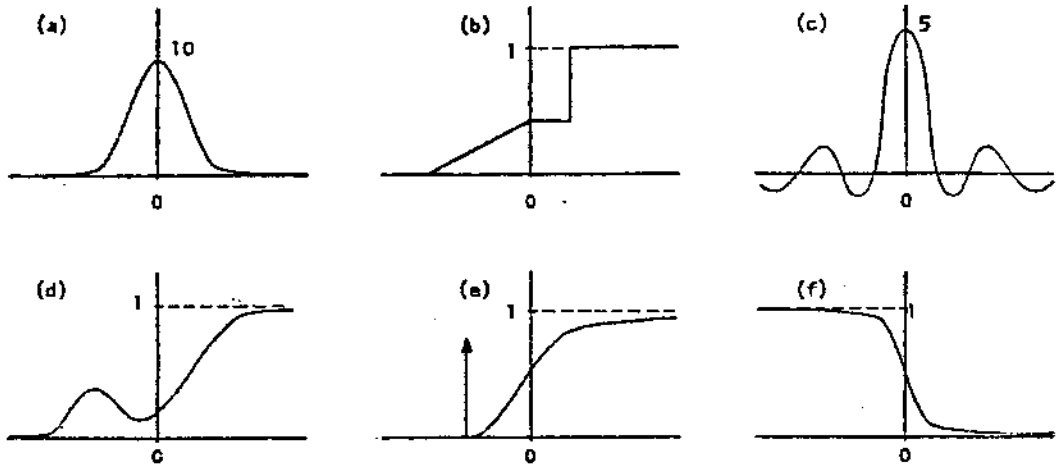
1). (12%)
For the following 3x4 matrix **A**, (a). find the 4x3 transpose matrix $\mathbf{B}=\mathbf{A}^T$; (b). By interchanging row 1 and row 3 of **B** to produce matrix **C**. Find the elementary matrix **E** such that **EB** = **C**.

$$A = \begin{matrix} 1 & -1 & 3 & 2 \\ 0 & 0 & 1 & 6 \\ 1 & -2 & 4 & 0 \end{matrix}$$

2). (14%)
Let a and k be positive numbers. Consider a pulse-modulated radar in which a high-frequency signal $k\cos\omega_0 t$ is switched on for a short time interval $(-a, +a)$ (relative to the time between pulses). Find the amplitude spectrum of such a pulse-modulated signal

$$g(t) = \begin{cases} k\cos\omega_0 t, & \text{if } -a < t < +a \\ 0, & \text{if } t > +a, \text{ or } t < -a. \end{cases}$$

3). (8%)
For each of the following sketches, indicate whether it could be a valid cumulative distribution function (cdf), or a valid probability density function (pdf), or either, or neither.



4). (16%)
Messages arrive at a computer at an average rate of 15 messages per second. The number of messages that arrive in 1 second is known to be a Poisson random variable.
(a). Find the probability that no messages arrive in 1 second.
(b). Find the probability that more than 10 messages arrive in a 1-second period. [Hint: Use $P_{k+1} = (\lambda/(k+1))P_k$ to compute the probabilities.]

(背面仍有題目,請繼續作答)

5). (15%)
Show that the eigenvalues of

$$\begin{bmatrix} \alpha & \beta & \gamma \\ \beta & \phi & \varepsilon \\ \gamma & \varepsilon & \theta \end{bmatrix}$$

are real if all the matrix elements are real numbers.

6). (10%)
Produce a matrix that diagonalizes the following given matrix, or show that this matrix is not diagonalizable.

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

7). (10%)
Find the following Laplace Transform. (a) $\boldsymbol{L}[1]$; (b) $\boldsymbol{L}[\sin 2t]$

8). (15%)
Use the Laplace transform to solve the following system.

$$x' + y = t$$
$$4x + y' = 0$$
$$x(0) = 1, \quad y(0) = 2$$