

1. Let  $X$  and  $Y$  be independent continuous random variables with probability density functions  $f_1$  and  $f_2$ , respectively. Find the probability density function of the random variable  $X+Y$ . (10%)
2. A fair coin is flipped repeatedly. What is the probability that the fifth tail occurs before the tenth head? (10%)
3. What is the probability of an even number of successes in  $n$  independent Bernoulli trials? (Bernoulli success probability is  $p$ ) (15%)
4. The number of lost days per month of drilling experiences by a sample of offshore platforms during 20 months are given below:
 

7	6	1	9	1	2	5	4	0	3
3	9	6	2	4	1	7	2	8	7

 Determine values for the following sample statistics (a) mean (b) median (c) mode (d) variance (e) coefficient of variation (15%)

(背面仍有題目,請繼續作答)

5. Suppose  $\det(\mathbf{A}) = 5$ . (a) Please determine the relation equation between  $\text{adj}\mathbf{A}$  and  $\mathbf{A}^{-1}$ , and (b) please determine the relation equation between  $\text{adj}(\text{adj}\mathbf{A})$  and  $\mathbf{A}$ . (6%, 10%)
6. Show that (a)  $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ , and (b) if  $\mathbf{A}$  is similar to  $\mathbf{B}$ , then  $\text{tr}(\mathbf{A}) = \text{tr}(\mathbf{B})$ , where  $\text{tr}$  presents *trace*. (10%, 6%)
7. (a) Let  $\mathbf{S}$  be a subspace of  $\mathbf{R}^n$ , please give the definition of the *orthogonal complement* of  $\mathbf{S}$ ,  $\mathbf{S}^\perp$ . (b) Please give the definition of the *direct sum*,  $\oplus$ . (c) Show that if  $\mathbf{S}$  is a subspace of  $\mathbf{R}^n$ , then  $\mathbf{R}^n = \mathbf{S} \oplus \mathbf{S}^\perp$ . (4%, 4%, 10%)