

1. Find the general solution of the following ordinary differential equation

$$y' + \frac{2x - 5y - 9}{4x - y - 9} = 0 \quad (17\%)$$

2. Solve the initial value problem

$$y'' + 9y = f(t); \quad y(0) = y'(0) = 1; \quad (18\%)$$

$$f(t) = 0 \quad \text{if } 0 \leq t < \pi, \quad f(t) = \cos(t) \quad \text{if } t \geq \pi.$$

$$3. A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 27 & -27 & 9 \end{bmatrix}$$

(a) Find the eigenvalues of  $A$ . (4%)

(b) Is  $A$  diagonalizable? Justify your answer. (6%)

(c). Compute  $e^{tA}$ , (10%)

$$\text{where } e^{tA} = I + \frac{t}{1!} A + \frac{t^2}{2!} A^2 + \frac{t^3}{3!} A^3 + \dots + \frac{t^n}{n!} A^n + \dots$$

4. Compute the complex Fourier series of  $f(x)$ . Note that  $f(x)$  has period 2,

$$\text{and } f(x) = \begin{cases} x^2 + x & \text{for } 0 \leq x < 1 \\ x^2 - x + 2 & \text{when } 1 \leq x < 2 \end{cases} \quad (15\%)$$

$$5. \text{ Evaluate } \int_0^\pi \frac{d\theta}{(a + \cos \theta)^2} \quad a > 1. \quad (15\%)$$

$$6. \text{ Given that } \oint_C \frac{f(z)}{z} dz = 2\pi i \quad \text{and} \quad \oint_C \frac{f(z)}{(z-1)^2} dz = 4\pi i.$$

If let  $f(z) = a + bz$ , find the  $a$  and  $b$ . (15%)