

1. (15%) Find the differential equation associated with the solution

$$y = C_1 e^{3x} + C_2 e^{2x} + C_3 e^x.$$

2. (20%) Solve $(2x - 5y + 3)dx - (2x + 4y - 6)dy = 0$.

3. (15%) Solve $p^4 - (x + 2y + 1)p^3 + (x + 2y + 2xy)p^2 - 2xyp = 0$, $p \equiv dy/dx$.

4. (14%) Let W be the subspace $sp(\sin 2x, \cos 2x)$ (where $sp(X)$ denotes the set of all linear combinations of vectors in X) of the vector space of all real-valued functions with domain \mathbf{R} , and let $B = (\sin 2x, \cos 2x)$. Find the matrix representation A relative to B for the linear transformation $T: W \rightarrow W$ defined by

$$T(f) = D^2(f) + 3D(f) + 2f, \text{ where } D \text{ presents the derivative operator.}$$

5. (10%, 10%) Let $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$. (a) Find the rank and eigenvalues of A . (b)

Calculate the eigenvectors corresponding to nonzero eigenvalues.

6. (16%) Derive and determine the inverse of the matrix $\begin{bmatrix} A & D \\ C & B \end{bmatrix}$, if A and

$B - CA^{-1}D$ are nonsingular.