1. The probability that an earthquake will damage a certain structure during a year is 0.015. The probability that a hurricane will damage the same structure during a year is 0.025. If the probability that both an earthquake and a hurricane will damage the structure during a year is 0.0073, what is the probability that next year the structure will not be damaged by a hurricane or an earthquake? (10%)

2. From a faculty of six professors, six associate professors, ten assistant professors, and twelve instructors, a committee, of size six is formed randomly. What is the probability that there is at least one person from each rank on the committee? (10%)

3. One prisoner is given 2N balls, which differ from each other only in that half of them are green and half are red. The king instructs the prisoner to divide the balls between two identical urns. One of the urns will then be selected at random, and the prisoner will be asked to choose a ball at random from the urn chosen. If the ball turns out to be green, the prisoner will be freed. How should he distribute the balls in the urn to maximize his chances of freedom? (15%)

4. Let X be a random variable with the probability density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty$$

Find the density function of Z= $\tan^{-1}$ X. (15%)

5. Suppose $A=\begin{bmatrix} 0 & 0 & 1 & -3 & 2 \\ 2 & -1 & 4 & 2 & 1 \\ 4 & -2 & 9 & 1 & 4 \\ 2 & -1 & 5 & -1 & 5 \end{bmatrix}$. (a) What is the rank of A? (b) What is the basis for the row space of A? (c) What is a basis for the column space of A? (d) True or false: Rows 1, 2, 3 of A are linearly independent. (e) What is the dimension of the left nullspace of A? (f) What is the general solution to $A\mathbf{x}=0$? $(6\times 3\%)$

6. Find the transition matrix representing the change of coordinates on P_3, where P_3 denotes the set of all polynomials of degree less than 3, from the ordered basis $\left[1,\ x,\ x^2\right]$ to the ordered basis $\left[1,\ 1+x,\ 1+x+x^2\right]$. (12%)

7. Let A be an $n\times n$ matrix with positive real eigenvalues $\lambda_1>\lambda_2>\cdots>\lambda_n$. Let $\mathbf{x}_i$ be an eigenvector belonging to λ_i for each i and let $\mathbf{x}=\alpha_1\mathbf{x}_1+\alpha_2\mathbf{x}_2+\cdots+\alpha_n\mathbf{x}_n$. (a) Show by induction that $A^m\mathbf{x}=\sum_{i=1}^{n}\alpha_i\lambda_i^m\mathbf{x}_i$. (b) If $\lambda_1=1$, find $\lim_{m\to\infty}A^m\mathbf{x}$. (12%, 8%)