

- The random process $Z(t)$ is defined by $Z(t) = X \cos(2\pi f_1 t + \theta) + Y \sin(2\pi f_2 t + \theta)$. Both X and Y are zero-mean Gaussian random variables with variances σ_x^2 and σ_y^2 . Random variable θ is uniformly distributed over $(0, 2\pi)$. And all these three random variables are independent.
 - Determine the ensemble average $m_z(t)$. (5%)
 - Determine the auto-correlation function $R_z(t_1, t_2)$. (5%)
 - Is $Z(t)$ wide-sense stationary (WSS)? (5%)
 - Determine and plot the two-sided power spectral density (PSD) of $Z(t)$. (5%)

- A sinusoidal message signal $x(t) = \cos 16000\pi t$ modulates the carrier $c(t) = 10 \cos 2\pi f_c t$, $f_c \gg 10\text{kHz}$. The modulation scheme is conventional AM and the modulation index is 0.5. The channel noise is AWGN with two-sided PSD = 10^{-10}W/Hz . At the receiver the signal is processed as shown in Figure 2(a). The frequency response of the bandpass filter is shown in Figure 2(b), where $B_T = \Delta f = 20\text{kHz}$.
 - Determine the signal power (in dBm) and the noise power (in dBm) at the output of the BPF. (10%)
 - Determine the $(S/N)_o$ (in dB) at the output. (10%)

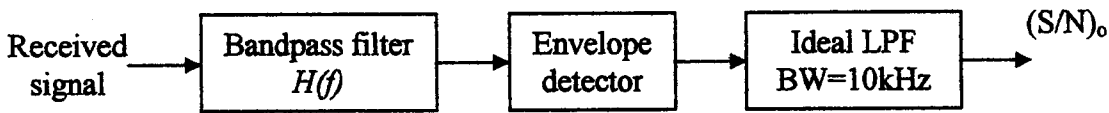


Figure 2(a)

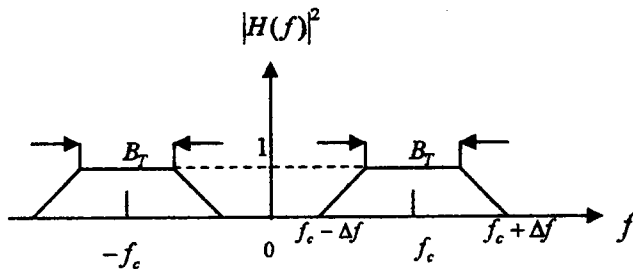


Figure 2(b)

- A message source has the alphabet $A = \{-5, -3, -1, 0, 1, 3, 5\}$ with corresponding probabilities $\{0.05, 0.05, 0.15, 0.1, 0.05, 0.25, 0.35\}$. This source is quantized according to the quantization rule

$$\begin{cases} q(-5) = q(-3) = 4 \\ q(-1) = q(0) = q(1) = 0 \\ q(3) = q(5) = -4. \end{cases}$$
 - Find the entropy of the quantized output. (5%)
 - Design a Huffman code for the quantized output. Determine the average codeword length and the coding efficiency. (5%)
 - Determine the signal to quantization noise ratio S_q/N_q (in dB), where S_q is the quantized signal power and N_q is the quantized noise power. (5%)

4. Consider the transmission of a message via BPSK signal over a bandpass AWGN channel with an ideal frequency response over $810\text{MHz} \leq f \leq 890\text{MHz}$ Hz and single sided PSD = 10^{-8} W/Hz.
- (a) What is the maximum transmitted data rate if null-to-null bandwidth is considered? (5%)
 - (b) If the raised cosine channel spectrum is desired, what is the transmitted data rate for roll-off factor $\alpha=33\%$ (or $1/3$). (5%)
 - (c) If data rate = 40Mbps and the required bit-error-rate is 10^{-5} , determine the minimum received signal power (in dBm) for coherent detection of the BPSK signal. (10%)
 - (d) If data rate = 60Mbps and received signal power = 37.8dBm, determine the bit-error-rate for non-coherent detection of the DEBPSK signal. (10%)
- (Note: 1. It is required $E_b/N_0 = 12.6$ dB for BFSK signal with coherent detection and $P_b = 10^{-5}$.
2. The bit-error-rate of BFSK modulation with non-coherent detection is $P_b = \frac{1}{2} \cdot \exp(-E_b/2N_0)$.)
5. The generator polynomial of a (7, 4) linear cyclic code is $g(x) = 1+x+x^3$.
- (a) Determine the output (in polynomial form) of the encoder if the input message polynomial $m(x) = 1+x^2$. (5%)
 - (b) If the received polynomial $r(x) = 1+x^3+x^5+x^6$ (i.e., $\underline{r} = (1001011)$), what is the decision of the error-detector at the receiver (error or no error)? Why? (10%)