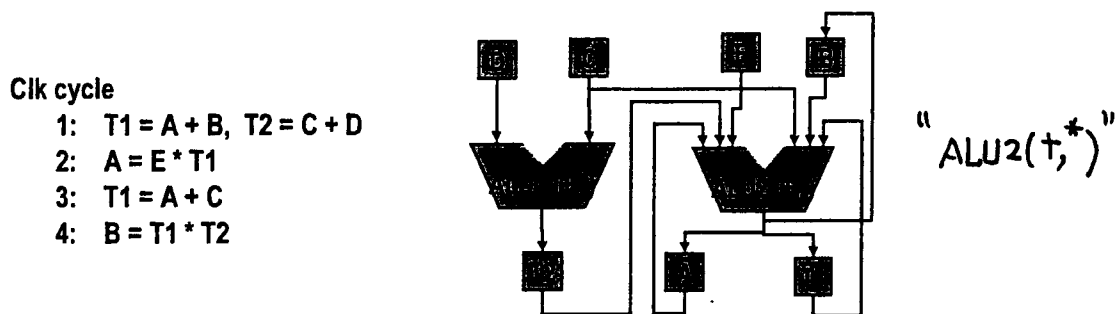


1. In the ASIC design, we have gotten the following scheduled simple operation description and its correspond datapath. Please find the minimal number of buses used for the datapath, and redraw the new datapath. 15%



2. Let A be a set with $|A| = n$, and let R be an equivalence relation on A with $|R| = r$. Is $r - n$ odd or even? Give your reason. 15%
3. Let $f: A \times A \rightarrow B$ be a binary operation on A . An element $x \in A$ is called an identity for f if $f(y, x) = f(x, y) = y$, for all $y \in A$. If $A = \{a, b, c, d, e\}$, how many closed binary operations on A have e as the identity? 15%
4. Consider a tennis tournament for n players, where $n = 2^k, k \in \mathbb{Z}^+$. In the first round $n/2$ matches are played, and the $n/2$ winners advance to round 2, where $n/4$ matches are played. This halving process continues until a winner is determined. For $n = 2^k, k \in \mathbb{Z}^+$, let $f(n)$ count the total number of matches played in the tournament. Please find and solve a recurrence relation for $f(n)$. 15%
5. How many ways can we to color 3 colors in the vertices of a cube? 20%
6. Let $G = (V, E)$ be an undirected connected loop-free graph. Suppose further that G is planar and determines 53 regions. If, for some planar embedding of G , each region has at least five edges in its boundary, prove that $|V| \geq 82$. 20%