1. A pyramid has a square base and 4 faces that are equilateral triangles. If we can move the pyramid in space, how many nonequivalent ways are there to paint its five faces, if we have paint of four different colors and the color of the base must be different from the colors of the triangular faces?   15%

2. Solve the recurrence equation $a_{n+2} - a_n = \sin(n\pi/2)$.   15%

3. In the VLSI hardware design, we have derived the following data-path and its corresponding scheduled operation description. Please find the minimal number of buses used for the datapath, and redraw the new data-path. (Hint: using graph coloring)   15%
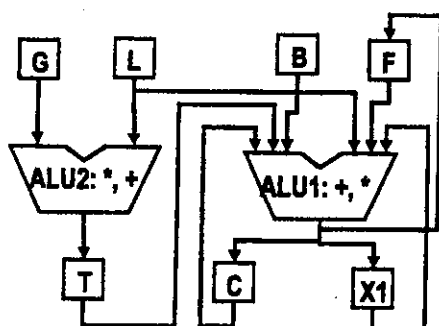
Clk cycle
1: X1 = C + L
2: C = B * X1
3: X1 = C + F, T = L + G
4: F = X1 * T

4. Please use the generating function to find the value of $3*2*1 + 4*3*2 + ... + (n+1)n(n-1)$?   15%

5. For $n \in Z^+$, let $u = \{1, 2, 3, ..., n\}$. Define the relation R on all subsets of u by ARB if $A \not\subseteq B$ and $B \not\subseteq A$. How many ordered pairs are there in this relation?   15%

6. For $A \in R^+$, define $\oplus$ and $\otimes$ by $a \oplus b = ab$ and $a \otimes b = a^{\log_2 b}$, $a, b \in A$.   (a) Is $(A, \oplus, \otimes)$ is a commutative ring? Give your reason,   (b) Is it an integral domain or a field? Give your reason.   10%

7. For a simple connected planar graph $G(V, E)$ with $|V| = 7$ and $|E| = 15$, how many edges does each region of G have? Give your reason.   15%