編號： 278　系所：電腦與通信工程研究所丙組　　科目：電磁數學

本試題是否可以使用計算機： ☐可使用 ， ☑不可使用 （請命題老師勾選）

1. (15%) Which of the following could be solved by separation of variables?

   (a) $u_{xy} + 2u = 0$　　(b) $x^2 u_{xx} + y u_{yy} = 0$

   (c) $2u_{xx} + 3u_{xy} - u_y = 0$

2. (20%) By using a trial solution $u(x,t) = X(x)T(t)$ and separation of variables, find solution of the following PDE:

   $$u_{xx} + u_{tt} = 0 \text{ where } u(0,t) = \sin t \text{ and } u(\infty,t) = 0$$

3. (15%) Find the general solution of the following PDE:

   $u_x + u = u_t$ **Hint**: change variables to $\varepsilon = x$, $\phi = -x - t$

4. (20%) Solve $A^2 \boldsymbol{x} = \boldsymbol{b}$ if $A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & -1 & 2 \\ -6 & 2 & 0 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} -2 \\ 14 \\ 12 \end{bmatrix}$.

5. (15%) Let $T: R^2 \to R^2$ be the linear transformation defined by

   $T([x, y]) = [4x - 2y, 2x + 3y]$. Find the area of the image in R^2 under T of the disk $(x-1)^2 + (y+2)^2 \le 9$.

6. (15%) For vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ in an inner product space, derive and find the sufficient and necessary condition such that $\boldsymbol{v} - \boldsymbol{w}$ and $\boldsymbol{v} + \boldsymbol{w}$ are perpendicular.