

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Let X_1, X_2, X_3, \dots be a sequence of independent and identical random variables, each with expectation μ and variance σ^2 . Prove that the distribution of $Z_n = \frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}}$ converges to the distribution of a standard normal random variable. (10%)
2. Let the joint probability density function of X and Y be given by $f(x, y) = \begin{cases} ye^{-y(1+x)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$. (a) Show that $E(X)$ does not exist. (5%) (b) Find $E(X|Y)$. (5%)
3. There are three types of animals in a laboratory: 15 type I, 13 type II, and 12 type III. Animals of type I react to a particular stimulus in 5 seconds, animals of types II and III react to the same stimulus in 4.5 and 6.2 seconds, respectively. A psychologist selects 10 of these animals at random and finds that exactly four of them react to this stimulus in 6.2 seconds. What is the probability that at least two of them react to the same stimulus in 4.5 seconds? (10%)
4. In data communications, a message transmitted from one end is subject to various sources of distortion and may be received erroneously at the other end. Suppose that a message of 64 bits (a bit is the smallest unit of information and is either 1 or 0) is transmitted through a medium. If each bit is received incorrectly with probability 0.0001 independently of the other bits, what is the probability that the message received is free of error? (10%)
5. The simplest error detection scheme used in data communication is parity checking. Usually messages sent consist of characters, each character consisting of a number of bits (a bit is the smallest unit of information and is either 1 or 0). In parity checking, a 1 or 0 is appended to the end of each character at the transmitter to make the total number of 1's even. The receiver checks the number of 1's in every character received, and if the result is odd it signals an error. Suppose that each bit is received correctly with probability 0.999, independently of other bits. What is the probability that a 7-bit character is received in error, but the error is not detected by the parity check? (10%)

(背面仍有題目, 請繼續作答)

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6. Solve the linear system $Az = \begin{bmatrix} -1+i \\ 2+i \\ 1 \end{bmatrix}$ if $A = \begin{bmatrix} i & 1-i & 1+i \\ 0 & 1 & i \\ 1-i & -i & 1-i \end{bmatrix}$. (20%)

7. Let $T: R^2 \rightarrow R^3$ be the linear transformation defined by $T([x, y]) = [2x+3y, x-y, 2y]$.

Find the area of the image in R^3 under T of the disk $x^2 + y^2 \leq 9$. (14%)

8. Is every Hermitian matrix normal? If yes, prove it. If no, give a counter example. (8%)

9. Is every unitary matrix normal? If yes, prove it. If no, give a counter example. (8%)