1. For p a prime determine all elements $a \in Z_p$ where $a^2 = a$. (10%)

2. Find the number of n-digit words generated from the alphabet {0, 1, 2, 3, 4} in each of which the total number of 0's and 1's is even. (15%)

3. Apply the state minimization process to the following machine. (15%)

	Next state		Output	
	0	1	0	1
S_1	S_6	S_3	0	0
S_2	S_3	S_1	0	0
S_3	S_2	S_4	0	0
S_4	S_7	S_4	0	0
S_5	S_6	S_7	0	0
S_6	S_5	S_2	1	0
S_7	S_4	S_1	0	0

4. On the first day of a new year, Joseph deposits $1000 in an account that pays 6% interest compounded monthly. At the beginning of each month he added $200 to his account. If he continues to do this for next four years (so that he makes 47 additional deposits of $200), how much will his account be worth exactly four years after he opened it? (15%)

5. In how many different ways can we use two different colors to paint the faces of a cube. (15%)

6. Let $f, g: \boldsymbol{Z}^+ \rightarrow \boldsymbol{R}$ where $f(n) = n^2 + n$ and $g(n) = (1/2)n^3$. Please prove that $f \in O(g)$ but $g \notin O(f)$. (15%)

7. Please find a 3 clock cycles scheduled data flow graph (the one like a state diagram) for the following computations and derive the minimum number of registers used in the graph using 2 adders and 1 multiplier. Assume both of the adder and the multiplier have one clock cycle delay. (Hint: using the graph coloring approach) (15%)

   $r = g + h + i$

   $s = g + c + h * c$