

系所組別： 電腦與通信工程研究所乙組

考試科目： 通信數學

考試日期：0307，節次：3

※ 考生請注意：本試題 可 不可 使用計算機

1. (10%) Let
- X
- be a discrete random variable with probability mass function

$$p(i) = \frac{6}{\pi^2 i^2}, \quad i = 1, 2, 3, \dots;$$

What is the moment-generating function of X ?

2. (15%) Let
- X
- and
- Y
- be independent (strictly positive) exponential random variables with parameter
- λ
- . Are the random variables
- $X + Y$
- and
- X/Y
- independent?

3. (15%) Let the probability density function of a random variable
- X
- be

$$f(x) = \frac{x^n}{n!} e^{-x}, \quad x \geq 0.$$

Show that

$$P(0 < X < 2n + 2) > \frac{n}{n + 1}.$$

4. (10%) Let
- $X_1, X_2, X_3,$
- and
- X_4
- be independent exponential random variables, each with parameter
- λ
- . Find
- $P(X_{(4)} \geq 3\lambda)$
- , where
- $X_{(k)}$
- is the
- k
- th order statistic.

5. (30%) Let
- A
- be an
- 5×5
- matrix with eigenvalues
- $1, -1, 2, -2, 4$
- .

- (a) Find the determinant of A^2 .
- (b) Is A an invertible matrix? (State the reason.)
- (c) Is A similar to a diagonal matrix? (State the reason.)
- (d) Find the trace of A . (The trace of A is the sum of the diagonal elements of A .)
- (e) Find the limit $\lim_{n \rightarrow \infty} A^n$ if it exists.

6. (20%) Consider a vector space of polynomials, defined by

$$V = \{p(t) = c_0 + c_1 t + c_2 t^2 \mid 0 \leq t \leq 1, c_0, c_1, c_2 \text{ are real numbers.}\}$$

Let T be a transformation with domain V , and $T(p(t)) = tp'(t) + 6 \int_0^1 p(t) dt$, where $p'(t)$ is the derivative of $p(t)$.

- (a) Is T a linear transformation?
- (b) Find the eigenvalues of T and give a corresponding eigenfunction for each eigenvalue.