※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. (10%) Use of residue methods to evaluate

$$\int_{-\infty}^{+\infty} \frac{\cos mx}{x^2+x+1}dx,\ m=1,2,3,...$$

2. (20%) Use of contour integral methods to evaluate

$$\int_{1}^{+\infty} \frac{1}{x\sqrt{x^2-1}}dx$$

3. (15%) Solve $x\frac{dy}{dx}+y=x^2y^2$

4. (20%) Solve the following differential equations.
   (a) $2y''-5y'-3y=0$
   (b) $y''-10y'+25y=0$

5. (20%) Define the Frobenius norm $\|A\|_F$ of an $m\times n$ matrix $A=[a_{ij}]$ as

$$\|A\|_F=\left(\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}^2\right)^{1/2}.\ \text{Let } B=\begin{bmatrix}5 & 8 & 10\\ 11 & 8 & -2\\ 13 & 4 & 2\\ 3 & 12 & 6\end{bmatrix}.$$

   (a) Find the matrix C of rank 1 that minimizes $\|B-C\|_F$.
   (b) Find the matrix D of rank 2 that minimizes $\|B-D\|_F$.

6. (15%) For the rectangular region $R=\{(x,y):0\le x\le 1, 0\le y\le 1\}$, solve the Dirichlet problem $\nabla^2 u(x,y)=0$, with the boundary conditions $u(0,y)=u(x,0)=0$, $u(x,1)=\pi$, and $u(1,y)=2\pi$.