

系所組別：微電子工程研究所，電機系甲乙丙丁戊組，電通所甲丁組

考試科目：工程數學

考試日期：0307，節次：3

※ 考生請注意：本試題 可 不可 使用計算機

1. Find $\max_z \left[\min_{\mathbf{x}^T \mathbf{z} = 0} \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}} \right]$, where \mathbf{x} and \mathbf{z} are vectors of dimension 4 and the matrix

$$\mathbf{A} = \begin{bmatrix} 5 & 0 & 1 & -2 \\ 0 & 5 & 2 & -1 \\ 1 & 2 & 5 & 0 \\ -2 & -1 & 0 & 5 \end{bmatrix}. \quad (20\%)$$

2. Find the series solution $y(x, t)$ of the following boundary value problem for $0 \leq x \leq 1$ and $t > 0$ by separation of variables. (You don't have to solve the coefficients numerically.) (15%)

$$\begin{aligned} \frac{\partial y(x, t)}{\partial t} &= 2 \frac{\partial^2 y(x, t)}{\partial x^2} \quad \text{for } 0 < x < 1, t > 0, \\ y(0, t) &= 0, \quad \left. \frac{\partial y}{\partial x}(x, t) \right|_{x=1} = -y(1, t) \quad \text{for } t > 0, \\ y(x, 0) &= x \quad \text{for } 0 \leq x \leq 1. \end{aligned}$$

3. Solve the initial-value problem $\frac{dy}{dx} = (-2x + y)^2 - 7$, $y(0) = 0$ (15%)

4. Solve the initial-value problem $\frac{d^2 y}{dx^2} + y = 4x + 10 \sin x$, $y(\pi) = 0$, $\frac{dy}{dx}(\pi) = 2$ (15%)

5. Classify all singularities of the function

$$f(z) = \frac{\pi + z + \sin z}{(1 + \cos z)^2} \quad (15\%)$$

6. Evaluate

$$\int_0^{\infty} \frac{dx}{x^2 + x + 1} \quad (20\%)$$

Hint: Consider the integral of $(\ln z)/(z^2 + z + 1)$