

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part I.

資料結構 (50%)

1. Answer **True** or **False** for the following statements. Give correct answers for **False** statements. (20%)

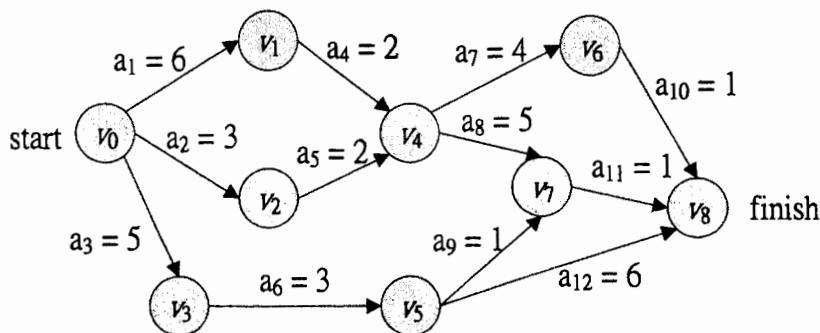
- (1) Let G be a graph with e edges and v vertices. If G is represented by adjacency lists, DFS requires $O(v^2)$ time.
- (2) If an AOV network represents a feasible project, its topological order is unique.
- (3) In static hashing, the worst-case number of comparisons needed for a successful search is $O(n)$ for open addressing. The number could be reduced to $O(\log n)$ by using chaining method.
- (4) Let d_i be the degree of vertex i in a graph G with $|V| = n$ and $|E| = e$, then $e = \sum_{i=0}^{n-1} d_i$.
- (5) The path from vertex u to vertex v on a minimal cost spanning tree of an undirected graph G is also a shortest path from u to v .

2. Given the following 8 runs: (15%)

7	10	23	2	6	4	16	12
13	18	28	15	11	20	17	21
22	29	32	19	24	30	31	25

- (1) Draw the corresponding winner tree.
- (2) Draw the restructured winner tree after one record has been output.
- (3) Draw the loser tree based on the answer of question (2).
- (4) Derive the total required time to merge n records through a winner tree with k runs.

3. Consider the following AOE network: (15%)



- (1) Obtain $e(i)$ and $l(i)$ for all activity i .
- (2) List all critical activities.
- (3) List all critical paths.

(背後仍有題目，請繼續作答)

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part II.

演算法 (50%)

1. Answer **True** or **False** for the following statements. Give correct answers for **False** statements. (20%)

- (1) If P equals NP, then NP equals NP-complete.
- (2) Any n -node unbalanced tree can be balanced using $O(\log n)$ rotations.
- (3) Let A_1 , A_2 , and A_3 be three sorted arrays of n real numbers (all distinct). In the comparison model, constructing a balanced binary search tree of the set $A_1 \cup A_2 \cup A_3$ requires $\Omega(n \log n)$ time.
- (4) Let P be a shortest path from some vertex s to some other vertex t in a graph. If the weight of each edge in the graph is increased by one, P remains a shortest path from s to t .
- (5) We can claim that there is a simpler way to reweight edges than the method used in Johnson's algorithm. Letting $w^* = \min_{(u,v) \in E} \{w(u,v)\}$, just define $\hat{w}(u,v) = w(u,v) - w^*$ for all edges $(u,v) \in E$.

2. Give a tight asymptotic upper bound (**O** notation) on the solution to the following recurrence. (10%)

$$T(n) = \begin{cases} 16T\left(\frac{n}{2}\right) + \Theta(1) & \text{if } n^2 > M, \\ M & \text{if } n^2 \leq M; \end{cases}$$

3. Prove that the longest increasing subsequence problem can be reduced to the edit distance problem. (10%)

Edit distance problem:

Input: Two sequences A and B and the cost for operations substitution (C_s), insertion (C_i), and deletion (C_d)

Output: The minimum cost sequence of edit operations to transform A into B.

Longest increasing subsequence problem:

Input: A sequence S.

Output: The longest sequence of positions $\{p_1, p_2, \dots, p_k\}$ such that $p_i < p_{i+1}$ and $S_i < S_{i+1}$.

4. Consider the linear-programming system with 9 different constraints $x_1 - x_5 \leq -5, x_1 - x_4 \leq -2, x_2 - x_1 \leq -3, x_2 - x_3 = 8, x_3 - x_1 \leq 5, x_3 - x_5 \leq 2, x_4 - x_3 \leq -3, x_5 - x_1 \leq 6, x_5 - x_4 \leq 1$, (1) Draw the constraint graph for these constraints. (2) Solve for the unknowns x_1, x_2, x_3, x_4 , and x_5 or explain if no solution exists. (10%)