系所組別：電機資訊學院-資訊聯招

考試科目：工程數學　　考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Given $A=\begin{bmatrix}1-p & q\\ p & 1-q\end{bmatrix}$, $0<p,q<1$, find A^n.

2. (10%) Given $A=LU=\begin{bmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ 2 & -2 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 0 & 2\\ 0 & 0 & m & m\\ 0 & 0 & m & m^2\end{bmatrix}$, $m\geq 0, m\in Z$

   (a) Please find the rank of A. (5%)

   (b) Do A and U have the same nullspace? Please explain it. (5%)

3. (30%) $A=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$, with real numbers *a,b,c,d,e,f,g,h,and i* and eigenvalue and eigenvector pairs

$$(1,\begin{bmatrix}0\\0\\1\end{bmatrix}),(-2,\begin{bmatrix}-1\\1\\0\end{bmatrix}),(-3,\begin{bmatrix}1\\1\\0\end{bmatrix})$$

   (a) Check if matrix A is singular, orthogonal, or positive definite, or not belonging to the above classes. (5%)

   (b) Find $\lim_{n\to\infty}(A^{-1})^n$ (5%)

   (c) Let $\mathbf{x}=\begin{bmatrix}p\\q\\r\end{bmatrix}, p,q,r\in real\ number$ be a unit vector, what is the maximum value of $\|A\mathbf{x}\|$ (5%)

   (d) Let $\mathbf{x}=\begin{bmatrix}p\\q\\r\end{bmatrix}, p,q,r\in real\ number$ be a unit vector, what is the minimum value of $\mathbf{x}^TA\mathbf{x}$ (5%)

   (e) What is the determinant of $3A+I$? I is the identity matrix. (5%)

   (f) What is the column space of $A+3I$? I is the identity matrix. (5%)

(背面仍有題目,請繼續作答)

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

4. (20%) Determine whether the given differential equation is exact, If it is exact, solve it, if not, explain why)

$(a).(2y-\frac{1}{x}+\cos 3x)\frac{dy}{dx}+\frac{y}{x^2}-4x^3+3y\sin x=0$

$(b).\ 3x^2ydx+(x^3-5)dy=0$

5. (10%) Solve the given differential equation by undetermined coefficients $y''+4y=6\sin(x)\cos(x)$.

6. (10%) Find the **particular solution** of the given high-order differential equation.

$$2\frac{d^5y}{dx^5}-7\frac{d^4y}{dx^4}+12\frac{d^3y}{dx^3}+8\frac{d^2y}{dx^2}=e^x(1+e^x(1+e^x(1+e^x)))$$

7. (10%) $y''+y=f(t)$, where $f(t)=\begin{cases}1, & 0\le t<\frac{\pi}{2}\\ \sin t, & t\ge\frac{\pi}{2}\end{cases}$ $y(0)=1,\ y'(0)=0$ Solve $y(t)$