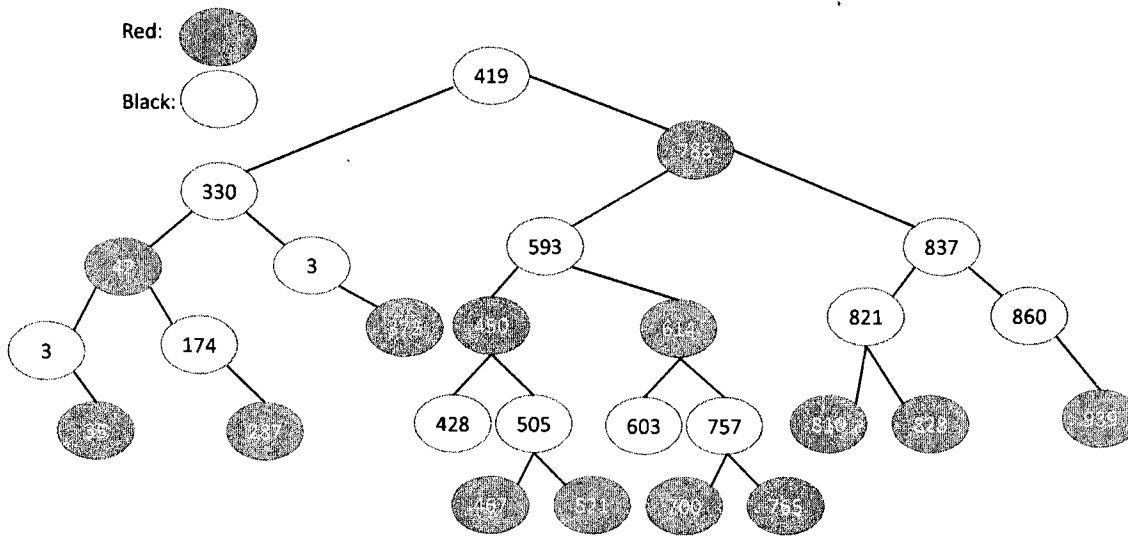


※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

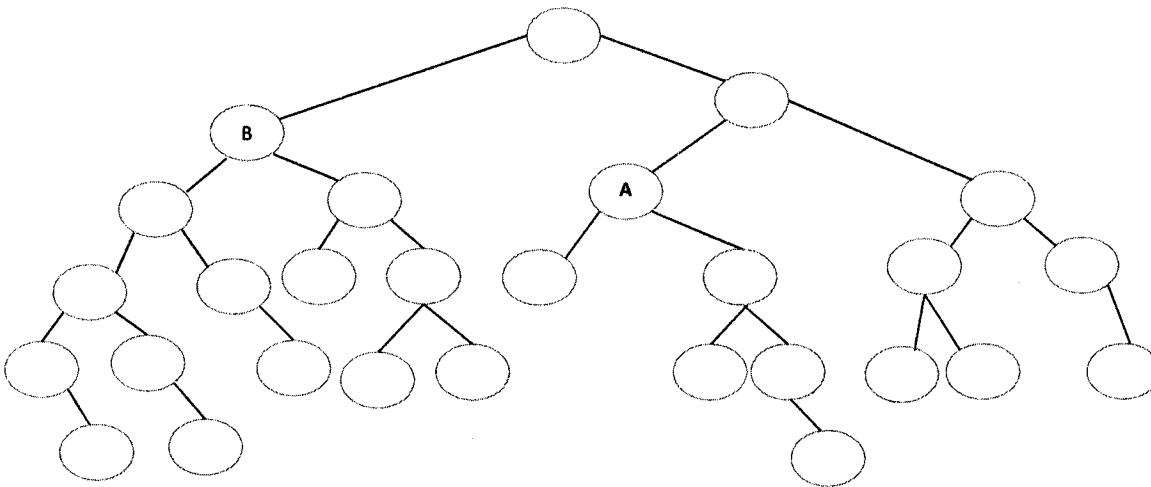
一、Data Structures (50%)

1. (20%)

Given a red-black tree in the following figure.



After one node with value 778 is inserted, the resulting red-black tree is shown as follows.



(a) (10%) Please describe the operation procedures for this insertion.

(b) (10%) Please indicate the color and value of node A and node B, respectively.

2. (10%) Given the expression, $(x+y)*w+u/(v+x*w)+z$, Please show the content in the stack after the operand v is read in postfix transformation.

3. (20%)

(a) (10%) Please finish the lost code for the **choose** function in the following Dijkstra shortest path implementation for a graph without negative-weight edges.

```

void shortestPath(int v, int cost[][MAX_VERTICES], int distance[], int n, short int found[])
{ /*cost is the adjacency matrix*/
    int i,u,w;
    for (i=0; i<n; i++) {
        found[i] = FALSE;
        distance[i] = cost[v][i];
    }
    found [v]= TRUE;
    distance[v]= 0;
    for (i=0; i<n-2; i++){
        u=choose(distance, n, found);
        found[u]= TRUE;
        for (w=0;w<n;w++)
            if (!found[w])
                if (distance[u]+cost[u][w] < distance[w])
                    distance[w]=distance[u]+cost[u][w];
    }
}

int choose (int distance[], int n, short int found[])
{
    int i, min, minpos;
    min = INT_MAX;
    minpos = -1;
    _____
    return minpos;
}

(b) (10%) In a directed graph without a cycle of negative length but with a negative-length edge, we can
implement Bellman-Ford algorithm as follows to compute shortest paths. Please fill in the lost code.
void BellmanFord (int n, int v){
    for (int i =0; i<n; i++)
        distance[i]=cost[v][i];
    for (int k=2; k<=n-1;k++)
        for ( _____ )
            for ( _____ )
                if (distance[u]> distance[i]+cost[i][u])
                    distance[u]=distance[i]+cost[i][u];
}

```

二、Algorithms (50%)

4. (10%) Prove or disprove: The single-source shortest paths problem can be solved in linear time in directed acyclic graphs.
5. (15%) The matrix-chain multiplication problem can be stated as follows: Given a chain $\langle A_1, A_2, \dots, A_n \rangle$ of n matrices, where for $i=1, 2, \dots, n$, matrix A_i has dimension $p_{i-1} \times p_i$, fully parenthesize the product $A_1 A_2 \dots A_n$ in a way that minimizes the number of scalar multiplications. Suppose that you have 6 matrices: A_1 has dimension 30×35 , A_2 has dimension 35×15 , A_3 has dimension 15×5 , A_4 has dimension 5×10 , A_5 has dimension 10×20 , A_6 has dimension 20×30 . Please calculate the minimum number of scalar multiplications.
6. (10%) Give asymptotic upper and lower bounds for $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$. Assume that $T(n)$ is constant for $n \leq 2$.
Make your bounds as tight as possible.
7. (15%) Consider the problem of finding the 5-vector $x = (x_i)$ that satisfies

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \leq \begin{pmatrix} 0 \\ -1 \\ 1 \\ 5 \\ -4 \\ -1 \\ -3 \\ -3 \end{pmatrix}$$

Determine whether there exists a solution or there is no solution.