

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) For $2xy'' + 5y' + xy = 0$, show that the indicial roots differ by an integer.

2. (20%) Solve the given differential equation. $4y'' - 4y' + y = e^{\frac{x}{2}} \sqrt{1-x^2}$

3. (20%)

Evaluate the given Laplace transform. Do not evaluate the integral before transforming.

$$\mathcal{L}\left\{t \int_0^t \tau e^{-\tau} d\tau\right\}$$

4. (20%) Solve $X^2 - 4X + 4I = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$

5. (20%) Let $V = \mathbb{R}^3$, and let $y_1 = (1, 1, 0)$, $y_2 = (2, 0, 1)$, and $y_3 = (2, 2, 1)$. Then $\{y_1, y_2, y_3\}$ is linearly independent. Please use

Gram-Schmidt orthogonalization process to compute the orthonormal vectors x_1, x_2, x_3 , and Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} = QR$,

what is the relationship between $\{x_1, x_2, x_3\}$ and Q ?