國立成功大學 104 學年度碩士班招生考試試題

系所組別：電機資訊學院-資訊聯招

考試科目：機率統計

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Pollution of the rivers in the United States has been a problem for many years. Consider the following events:

   A: the river is polluted,

   B: a sample of water tested detects pollution,

   C: fishing is permitted.

   Assume $P(A) = 0.3$, $P(B|A) = 0.75$, $P(B|A') = 0.20$, $P(C|A \cap B) = 0.20$, $P(C|A' \cap B) = 0.15$, $P(C|A \cap B') = 0.80$, and $P(C|A' \cap B') = 0.90$.

   (1) Find P(A ∩ B ∩ C). (5%) (hint: $P(A \cap B \cap C)=P(C|A \cap B)P(B|A)P(A)$)

   (2) Find P(B′ ∩ C). (5%) (hint: $P(B' \cap C)=P(A \cap B' \cap C) + P(A' \cap B' \cap C)$)

   (3) Find P(C). (5%)

   (4) Find the probability that the river is polluted, given that fishing is permitted and the sample tested did not detect pollution. (5%)

2. Suppose the manufacturer's specifications for the length of a certain type of computer cable are 2000 ± 10 millimeters. In this industry, it is known that small cable is just as likely to be defective (not meeting specifications) as large cable. That is, the probability of randomly producing a cable with length exceeding 2010 millimeters is equal to the probability of producing a cable with length smaller than 1990 millimeters. The probability that the production procedure meets specifications is known to be 0.99.

   (1) What is the probability that a cable selected randomly is too large? (10%)

   (2) What is the probability that a randomly selected cable is larger than 1990 millimeters? (10%)

3. In an NBA (National Basketball Association) championship series, the team that wins four games out of seven is the winner. Suppose that teams A and B face each other in the championship games and that team A has probability 0.55 of winning a game over team B.

   (1) What is the probability that team A will win the series in 6 games? (10%)

   (2) What is the probability that team A will win the series? (10%)

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4. A machine produces metal pieces that are cylindrical in shape. A sample of these pieces is taken and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimeters. Use those data to calculate two interval types. For all computations, assume an approximately normal distribution. The sample mean and standard deviation for the given data are $\bar{x} = 1.0056$ and $s = 0.0246$.
(Hint:

$T = \frac{\bar{X}-\mu}{s/\sqrt{n}}$, where $\bar{X}$: sample mean, μ: population mean, s: standard deviation, n: number of samples.

$P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1 - \alpha$, which we find an area of $\alpha/2$ under the normal curve.)

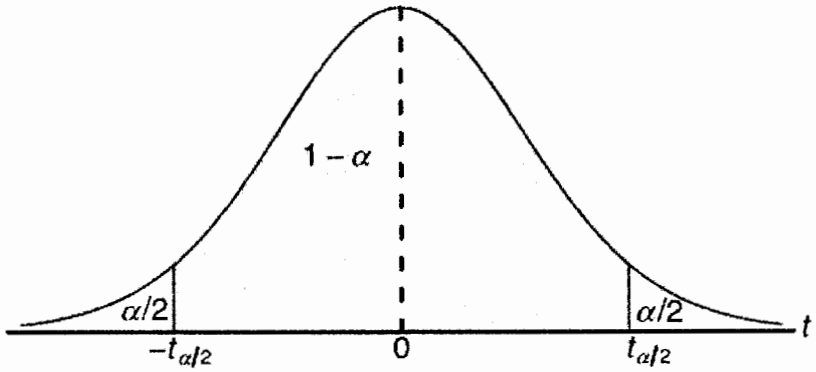

Figure 9.5: $P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1 - \alpha$.

(1) Based on equations at hint, please derive μ to be as $A < \mu < B$, find A and B. (10%)

(2) Find a 99% confidence interval on the mean diameter. (10%)

5. The joint density for the random variables (X, Y), where X is the unit temperature change and Y is the proportion of spectrum shift that a certain atomic particle produces, is

$$f(x,y) = \begin{cases} 10xy^2, & 0 < x < y < 1. \\ 0, & elsewhere. \end{cases}$$

(1) Find the marginal densities $g(x)$, $h(y)$, and the conditional density $f(y|x)$. (10%)

(2) Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased by 0.25 unit. (10%)

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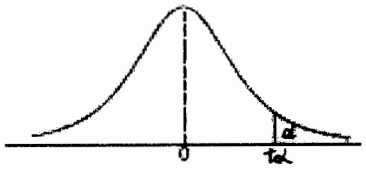

Table A.4 (continued) Critical Values of the t-Distribution

	α						
v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290