

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Note:

A. Please specify clearly "the name of a probability distribution" and "its parameters" that you use to calculate probability values.

B. Please find out probability "values" using attached tables whenever possible.

1. A producer of a certain type of electronic component ships to suppliers in lots of twenty. Suppose that 60% of all such lots contain no defective components, 30% contain one defective component, and 10% contain two defective components. A lot is picked, two components from the lot are randomly selected and tested, and neither is defective. What is the probability that zero defective component exists in the lot? **[10%]**

2. The joint probability density function of the random variable X, Y, and Z is

$$f(x, y, z) = \begin{cases} kxy^2z, & 0 < x, \quad y < 1, \quad 0 < z < 2, \\ 0, & \text{elsewhere} \end{cases}$$

2.1 Please find the constant k. **[10%]**

2.2 Please find  $P(X < \frac{1}{4}, Y > \frac{1}{2}, 1 < Z < 2)$ . **[10%]**

3. A production process produces electronic component parts. It is presumed that the probability of a defective part is 0.1. During a test of this presumption, 20 parts are sampled randomly and 5 defectives are observed.

3.1 Please find the probability that at least 5 defectives are observed in 20 randomly sampled parts. **[10%]**

3.2 From 3.1, please comment on the presumption that the defective rate is 0.1. **[10%]**

4. A random variable X has the Poisson distribution  $p(x; \mu) = e^{-\mu} \mu^x / x!$  for  $x = 0, 1, 2, \dots$

4.1 Please derive the moment-generating function of X. **[10%]**

4.2 Please use the moment-generating function in 4.1 to find the mean and the variance of X. **[10%]**

5. The following data represent the length of life, in seconds, of 50 fruit flies subject to a new spray in a controlled laboratory experiment:

17 20 10 9 23 13 12 19 18 24  
12 14 6 9 13 6 7 10 13 7  
16 18 8 13 3 32 9 7 10 11  
13 7 18 7 10 4 27 19 16 8  
7 10 5 14 15 10 9 6 7 15

5.1 Given that quantile factor is calculated as  $f_i = \frac{i-0.5}{n}$ , find the lower quartile and the upper quartile of the sample. [10%]

5.2 Given that an observation is regarded as an outlier if its distance from the box exceeds 1.5 times the interquartile range, construct a box-and-whisker plot of the sample. Mark the values of both lower and upper bounds of the whiskers. Mark the values of outliers (if any). [10%]

6. A machine produces metal pieces that are cylindrical in shape. A set of sample is drawn as the following: 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 1.01, 1.03, 0.99, 1.00, 1.00, 0.99, 0.98, 1.01, 1.02, 0.99 centimeters. The mean and standard deviation of the sample are 1.0025 centimeters and 0.0202 centimeter, respectively. For all computations, assume an approximately normal distribution. Please compute a 95% confidence interval on the mean diameter of all metal pieces produced by the machine. [10%]



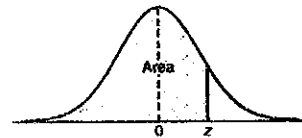


Table A.3 Areas under the Normal Curve

| <i>z</i> | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -3.4     | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| -3.3     | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| -3.2     | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| -3.1     | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| -3.0     | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| -2.9     | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| -2.8     | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| -2.7     | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| -2.6     | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| -2.5     | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| -2.4     | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| -2.3     | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| -2.2     | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| -2.1     | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| -2.0     | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| -1.9     | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| -1.8     | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| -1.7     | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| -1.6     | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| -1.5     | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| -1.4     | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| -1.3     | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| -1.2     | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| -1.1     | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| -1.0     | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| -0.9     | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| -0.8     | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| -0.7     | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| -0.6     | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| -0.5     | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| -0.4     | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| -0.3     | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| -0.2     | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| -0.1     | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| -0.0     | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |



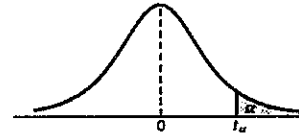


Table A.4 Critical Values of the *t*-Distribution

| <i>v</i> | $\alpha$ |       |       |       |       |       |        |
|----------|----------|-------|-------|-------|-------|-------|--------|
|          | 0.40     | 0.30  | 0.20  | 0.15  | 0.10  | 0.05  | 0.025  |
| 1        | 0.325    | 0.727 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 |
| 2        | 0.289    | 0.617 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303  |
| 3        | 0.277    | 0.584 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182  |
| 4        | 0.271    | 0.569 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776  |
| 5        | 0.267    | 0.559 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571  |
| 6        | 0.265    | 0.553 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447  |
| 7        | 0.263    | 0.549 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365  |
| 8        | 0.262    | 0.546 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306  |
| 9        | 0.261    | 0.543 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262  |
| 10       | 0.260    | 0.542 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228  |
| 11       | 0.260    | 0.540 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201  |
| 12       | 0.259    | 0.539 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179  |
| 13       | 0.259    | 0.538 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160  |
| 14       | 0.258    | 0.537 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145  |
| 15       | 0.258    | 0.536 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131  |
| 16       | 0.258    | 0.535 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120  |
| 17       | 0.257    | 0.534 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110  |
| 18       | 0.257    | 0.534 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101  |
| 19       | 0.257    | 0.533 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093  |
| 20       | 0.257    | 0.533 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086  |
| 21       | 0.257    | 0.532 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080  |
| 22       | 0.256    | 0.532 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074  |
| 23       | 0.256    | 0.532 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069  |
| 24       | 0.256    | 0.531 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064  |
| 25       | 0.256    | 0.531 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060  |
| 26       | 0.256    | 0.531 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056  |
| 27       | 0.256    | 0.531 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052  |
| 28       | 0.256    | 0.530 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048  |
| 29       | 0.256    | 0.530 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045  |
| 30       | 0.256    | 0.530 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042  |
| 40       | 0.255    | 0.529 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021  |
| 60       | 0.254    | 0.527 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000  |
| 120      | 0.254    | 0.526 | 0.845 | 1.041 | 1.289 | 1.658 | 1.980  |
| $\infty$ | 0.253    | 0.524 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960  |