國立成功大學 105 學年度碩士班招生考試試題

系　所：電機資訊學院-資訊聯招

考試科目：計算機數學

考試日期：0227，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Give a geometric explanation of why a homogeneous linear system consisting of two equations in three unknowns must have infinitely many solutions. (5%)

2. For each of the statements that follow, answer true if the statement is always true and false otherwise. In the case of a true statement, explain or prove your answer. In the case of a false statement, give an example to show that the statement is not always true. Assume that all the given matrices are $n \times n$. (10%)
   (a) $\det(AB) = \det(BA)$
   (b) $\det(A+B) = \det(A) + \det(B)$
   (c) $\det(cA) = c \det(A)$
   (d) $\det((AB)^T) = \det(A) \det(B)$
   (e) $\det(A) = \det(B)$ implies $A = B$

3. Determine whether the following are subspaces of $R^{2\times 2}$: (10%)
   (a) The set of all 2×2 lower triangular matrices
   (b) The set of all 2×2 matrices A such that $a_{11} = 1$
   (c) The set of all 2×2 matrices B such that $b_{11} = 0$
   (d) The set of all symmetric 2×2 matrices
   (e) The set of all singular 2×2 matrices

4. Let $\boldsymbol{A}$ be an 8×5 matrix of rank 3, and let $\mathbf{b}$ be a nonzero vector in $N(\boldsymbol{A}^T)$
   (a) Show that the system $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ must be inconsistent. (5%)
   (b) How many least squares solutions will there be to the system $\boldsymbol{A}\mathbf{x} = \mathbf{b}$? Explain. (5%)

5. For each statement that follows, answer true if the statement is always true and false otherwise. In the case of a true statement, explain or prove your answer. In the case of a false statement, give an example to show that the statement is not always true. (15%)
   (a) If A is an $n \times n$ matrix, then A and A^T have the same eigenvectors.
   (b) If A has eigenvalues of multiplicity greater than 1, then A must be defective.
   (c) If A is a 4×4 matrix of rank 3 and $\lambda = 0$ is an eigenvalue of multiplicity 3, then A is diagonalizable.
   (d) The rank of an $n \times n$ matrix A is equal to the number of nonzero eigenvalues of A, where eigenvalues are counted according to multiplicity.
   (e) If A is symmetric positive definite, then A is nonsingular and A^{-1} is also symmetric positive definite.

6. For $n \geq 1$, let a_n count the number of ways to write n as an ordered sum of odd positive integers. (For example, $a_4 = 3$ since $4 = 3+1 = 1+3 = 1+1+1+1$.) Find and solve a recurrence relation for a_n. (15%)

7. (a) How many bit strings of length 8 do not contain three consecutive 1s? (10%) (b) What is the probability that a ternary sequences (sequence made up of the digits 0, 1, 2) of length 8 contains even number of 1's bits? (10%)

8. Let p, q be two distinct primes. We denote relation x**R**y if x divides y. Under this relation **R**, please determine (a) the Hasse diagram of all positive divisors of p^2q^2, (b) glb$\{pq, p^2\}$, (c) How many edges are there in the Hasse diagram of all positive divisors of p^mq^2, $m \in Z^+$. (15%)