編號： 212

國立成功大學 107 學年度碩士班招生考試試題

系　　所：電機資訊學院-資訊聯招

考試科目：計算機數學　　考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。只有答案，沒有計算過程或說明原因，也不予計分。考卷含線性代數及離散數學

一、線性代數 (50%)

1. (25%) Let $A=\begin{bmatrix} 2 & 5 & 4 \\ 6 & 3 & 0 \\ 6 & 3 & 0 \\ 2 & 5 & 4 \end{bmatrix}$, R: range , N: nullspace

   (a) (5%) Find the singular value decomposition of A.

   (b) (5%) Find the orthonormal bases of $R(A^T)$.

   (c) (5%) Find the orthonormal bases of R(A).

   (d) (5%) Find the orthonormal bases of N(A).

   (e) (5%) Find the orthonormal bases of $N(A^T)$.

2. (10%) Let $M=\begin{bmatrix} a & 0 & a \\ a & 2a & -2a \\ a & a & 0 \end{bmatrix}$, where a=1.

   (a) (3%) Find the minimal polynomial of M

   (b) (3%) Find the Jordan canonical form of M.

   (c) (4%) Find the Jordan basis for M.

3. (10%) Let $M=\begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$

   (a) (5%) Whether matrix M is positive definite? Why?

   (b) (5%) Are all eigenvalues of M greater than 0 and smaller than 2? Why?

4. (5%) Let $M=\begin{bmatrix} -2 & 1 & -1 \\ 0 & 2 & 1 \\ -4 & 2 & 2 \\ 0 & 4 & 0 \end{bmatrix}$ and $b=\begin{bmatrix} -5 \\ 5 \\ 5 \\ -10 \end{bmatrix}$

   Please find a vector P such that P is in the column space of M and b-P is orthogonal to every vector in the column space of M.

二. Discrete Mathematics (TOTAL: 50%)(No score if you give no details.)

5. Suppose we randomly choose nonnegative integers x_1, x_2, x_3, and x_4 that solve the equation $x_1+x_2+x_3+x_4 = 10$. We assume that each solution has an equal probability of being chosen. Given that at least one of x_1 and x_2 is equal to 2, what is the probability that $x_2 = 2$? (15%)

6. Suppose that Mark selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains 5 red balls and 4 blue balls, and the second box contains 3 red balls and 6 blue balls. What is the probability that Mark picked a ball from the second box if he has selected a red ball? (15%)

7. Solve the following recurrence relation:

$$3a_n - 6a_{n-1} - 3a_{n-2} + 6a_{n-3} = 0$$

with $a_0 = 1, a_1 = 0, a_2 = 7$. (10%)

8. Find the set of all solutions x to the system of congruences: (10%)

$$x \equiv 4 (\text{mod } 5) \text{ and } x \equiv 5 (\text{mod } 15)$$