編號： 203 國立成功大學 109 學年度碩士班招生考試試題

系　　所：電機資訊學院-資訊聯招

考試科目：工程數學　　考試日期：0210，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) The general first-order differential equation form is M(x,y)dx+N(x,y)dy=0

If $M(x,y)=x^2\cos x-y(x\tan x-1), \mathrm{N}(\mathrm{x},\mathrm{y})=-x$

(a) Is the differential equation exact? Please explain the reason.

(b) Please find the general solution y.

2. (10%) Please solve the following differential equation.

$y'+p(x)y=r(x)y^n$, n>=2

3. (10%) Please solve the following differential equation

$4y''+36y=36\csc 3x$

4. (10%) Find the Laplace transform of y(t).

$y(t)=\cos(t-\frac{\pi}{4})$

5. (10%) Find the inverse Laplace transform of F(s)

$F(s)=\frac{s}{(s+1)(s-2)^2}$

6. (10%) Solve the following coupled differential equation.

$x''+(\alpha+\beta)\mathrm{y}'-\alpha\beta x=0$

$\mathrm{y}''-(\alpha+\beta)x'-\alpha\beta y=0,\ \alpha>0,\beta>0,\alpha\neq\beta$

$x(0)=0, x'(0)=1$

$y(0)=0, y'(0)=0.$

7. (20%) Assume we have collected the data from four types of sensors four times from two experiments. Put these data in the form of matrix A and B.

$$A=\begin{bmatrix}-7&0&5&0\\0&1&1&0\\-4&0&2&0\\0&0&0&0\end{bmatrix}\quad B=\begin{bmatrix}8&-7&1&0\\0&1&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix}$$

Check if matrix A and matrix B are diagonalizable. If it is diagonalizable, compute a matrix P such that $P^{-1}AP$ or $P^{-1}BP$ is a diagonal matrix and compute the determinant of A^4-5A^2+2I or B^4-5B^2+2I. I is an identity matrix.

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8. (20%) Let R and S be vector spaces with ordered bases A=$\left\{\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix},\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\}$ and

B=$\left\{1, x, 1+x^2, x+x^2\right\}$, respectively. Let T: R->S be a linear transformation such that

$T\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\right)=(c+d)x^2+(b+d)x+(a+c)$.

(a) What is the coordinate vector of $\begin{bmatrix}3 & 2\\ 4 & 5\end{bmatrix}$ with respect to A?

(b) Find a basis for the image space of T.

(c) Find a basis for the null space of T.

(d) What is the matrix form of T from A to B?