國立成功大學

110學年度碩士班招生考試試題

編　號： 204

系　所： 電機資訊學院-資訊聯招

科　目： 計算機數學

日　期： 0202

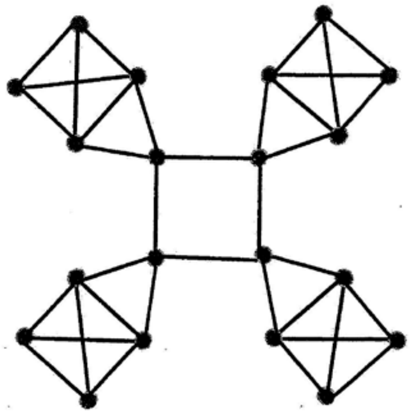
節　次： 第 3 節

備　註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、Discrete Mathematics (50%) (請說明如何求解過程,只寫答案不予計分)

1. (10%) (a) Assume that each vertex has a unique id, how many different spanning trees in the following figure? (5%)



● :vertex　　—— :edge

(b) We can show that the set {(x,y) | x,y are positive integers} is countable by finding a 1-1 mapping from (x,y) to an integer. Given (1,1) -> 1, (1,2)->2, (1,3)->4, (1,4)->7, (2,1)-> 3, (2,2)->5, (2,3)->8, (3,1)->6, (3,2)->9, (4,1)->10, please find which (x,y) maps to 465? (5%)

2. (10%) (a) Please prove that $x^5-2x^4+x^3-2x^2-7x-1=0$ has a solution x which is an integer. (5%)

(b) Given a 4x82 grid, each vertex is colored in one of three different colors, please show that we can always find a rectangle with four vertices in the same color. (5%)

3. (30%) (a) Assume N is a positive integer, please show that the statement "if $2^N -1$ is prime, then N is prime" is true using proof by contrapositive. (10%)

(b) 5*n+7*m=N, where n,m,N are integers and n≥0, m≥0, N≥24. Please show that we can always find n, m to satisfy the equality. For example, (n,m)=(2,2) corresponds to 5*2+7*2=24 and (n,m)=(5,0) corresponds to 5*5+7*0=25. (10%)

(c)Assume a set G whose elements are real numbers and the size of G is equal to 2^k, where k is a positive integer. We just want to find the maximum value and the minimum value in this set G and develop an algorithm in the following.

```
FindMaxMin(G)
 If G contains only two numbers, then compare these two numbers,
set M to be the larger one, and set m to be the smaller one,
 Else
   Divide G into two subsets with equal size G1 and G2
   Apply FindMaxMin(G1) to get M1 and m1
   Apply FindMaxMin(G2) to get M2 and m2
   M=max(M1,M2), m=min(m1,m2)
 Return M,m
```

We use T(N) to represent the number of comparisons when the set size is equal to N. $N=2^k$. Please find T(N) in terms of N. (10%)

國立成功大學 110 學年度碩士班招生考試試題

系　所：電機資訊學院-資訊聯招

考試科目：計算機數學

考試日期：0202，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

二. Linear Algebra (50%)

4. (10%)
Find the curve $y = C(-2)^x + D(-1)^{x+1}$, which gives the least squares fit to points $(x, y) = (0,0), (1,4), (2,6)$.

5. (30%. 15 pts each)

(a) Given $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find the Gram-Schmidt QR decomposition of $\mathbf{A}$, then use that decomposition to solve the least squares problem. (15%)

$$\mathbf{Ax} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

(b) Given $\mathbf{A} = \begin{bmatrix} 10 & 5 \\ -11 & 2 \\ -2 & 14 \end{bmatrix}$. Find the pseudoinverse of $\mathbf{A}$. (15%)

6. (10%)
Two eigenvectors of this circulant matrix $\mathbf{C}$ are $(1, i, i^2, i^3)^\top$ and $(1, i^2, i^4, i^6)^\top$. What are the eigenvalues λ_0 and λ_1?

$$\begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix} \begin{bmatrix} 1 \\ i \\ i^2 \\ i^3 \end{bmatrix} = \lambda_0 \begin{bmatrix} 1 \\ i \\ i^2 \\ i^3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & c_0 & c_1 & c_2 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & c_2 & c_3 & c_0 \end{bmatrix} \begin{bmatrix} 1 \\ i^2 \\ i^4 \\ i^6 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ i^2 \\ i^4 \\ i^6 \end{bmatrix}$$