

國立成功大學

110學年度碩士班招生考試試題

編 號： 205

系 所： 電機資訊學院-資訊聯招

科 目： 機率統計

日 期： 0202

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

**Note:**

1. When applying any probability distribution, please specify clearly the "name of the probability distribution" you used and "its parameters" to calculate probability values.

2. Please find out probability "values" using attached tables whenever possible.

**1.**

If  $X$  is a discrete random variable with uniform distribution, where  $g(x) > 0$  when  $x = -1, 0, 1$ , and  $2$  (i.e.,  $g(x) = 0$ , elsewhere). If  $Y$  is another discrete random variable where  $h(0) = 0.2$ ,  $h(1) = 0.3$ , and  $h(2) = 0.5$  (i.e.,  $h(y) = 0$  elsewhere). In addition,  $X$  and  $Y$  are independent. Please find the probability distribution of  $(XY + 3)$  and plot it. [15%]

2. The joint probability density function of the random variables  $X$ ,  $Y$ , and  $Z$  is:

$$f(x, y, z) = \begin{cases} \frac{4}{9}xyz^2, & 0 < x < 1, 0 < y < 1, 0 < z < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

2.1 Please find  $P(0 < Z < 2 \mid X = \frac{1}{2}, Y = \frac{1}{2})$ . [10%]

2.2 Please find  $E(XYZ + X^2Y^2 + \frac{Z}{9})$ . [10%]

**3. Normal distribution**

3.1  $X$  is a random variable following normal distribution  $f(x)$  with its parameters  $\mu$  and  $\sigma$ . Please prove that  $E(X) = \mu$  and  $E[(X - \mu)^2] = \sigma^2$ . [10%]

3.2 If  $X_1, X_2, X_3, \dots, X_n$  are independent and identically distributed normal random variables with mean of  $\mu$  and variance of  $\sigma^2$ . A new random variable  $Y = (X_1 + X_2 + X_3 + \dots + X_n)/n$ , please derive  $Y$ 's probability distribution  $h(y)$ ,  $Y$ 's mean, and  $Y$ 's variance. [10%]

3.3 Following 3.1,  $\bar{x}$  is the mean of a random sample of size  $n$  from a normal population with mean of  $\mu$  and variance of  $\sigma^2$ . Please derive a two-sided 95% confidence interval for population mean  $\mu$ . [10%]

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**4.**

A LCD manufacturer wish to examine the defective rate of its new production line. It is conjectured (推測) that the defective rate of this new production line is 40%. If 14 LCDs are randomly selected and 10 are found to be defective. Let  $X$  be the number of defective LCDs among 14, please find  $P(X \geq 10)$  and use the result to comment on the conjecture on the defective rate. [15%]

**5.**  $X$  is a binomial random variable with parameters of  $n, p$  and  $q=(1-p)$ .

**5.1** Please derive the moment generating function of  $X$ . [10%]

**5.2** Please use the moment generating function in 5.1 to derive mean and variance of  $X$ . [10%]

(Tables are excerpted from Walpole et al., Probability and Statistics for Engineers and Scientists, 9<sup>th</sup> edition.)

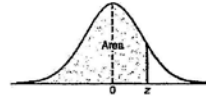


Table A.3 Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

