國立成功大學

111學年度碩士班招生考試試題

編　號：202

系　所：電機資訊學院-資訊聯招

科　目：計算機數學

日　期：0219

節　次：第 3 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、離散數學 (50%)

1. (10 points) How many bit string of length 20 contain at most Nineteen 1s? Please expressed as $A^B + C$

   (a) A =______ (3 point)

   (b) B =______ (3 points)

   (c) C =______ (4 points)

2. (10 points) Let (u,v) be a minimum-weight edge in a connected graph G. Show that (u,v) belongs to some minimum spanning tree of G.

3. (10 points) A={5,6,7,8} 、B={u,v} 、C={}

   consider a = |Power Set(A)|

   b = |A×B| (× is cross product)

   c = |Power Set(C)|

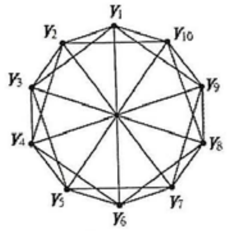
   Please calculate $2^c + b + a$.

4. (a) (5 points) How many quadrilateral are determined by the vertices of a regular polygon of n sides? ($n \geq 4$)

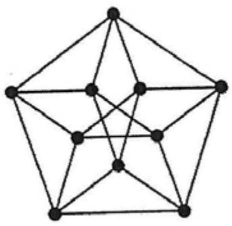
   (b) (5 points) How many if no side of the polygon is to be a side of any quadrilateral? ($n \geq 8$)

   Note: quadrilateral (四邊形)

5. (10 points) Calculate the chromatic number of figure(a) (5 points) and figure(b) (5 points). (not need to explain)



figure(a)



figure(b)

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二. Linear Algebra (50%)

6. Let $\mathbf{A}$ be a Hermitian matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ and orthonormal eigenvectors $\mathbf{u}_1, \ldots, \mathbf{u}_n$. For any nonzero vector $\mathbf{x}$ in $\mathbb{R}^n$, the *Rayleigh quotient* $\rho(\mathbf{x})$ is defined by

$$\rho(\mathbf{x}) = \frac{\langle \mathbf{A}\mathbf{x}, \mathbf{x} \rangle}{\langle \mathbf{x}, \mathbf{x} \rangle} = \frac{\mathbf{x}^H \mathbf{A} \mathbf{x}}{\mathbf{x}^H \mathbf{x}}$$

   (a) If $\mathbf{x} = c_1\mathbf{u}_1 + \cdots + c_n\mathbf{u}_n$, show that

$$\rho(\mathbf{x}) = \frac{|c_1|^2\lambda_1 + |c_2|^2\lambda_2 + \cdots + |c_n|^2\lambda_n}{\|\mathbf{c}\|^2}$$

   (10%)

   (b) Show that $\lambda_n \leq \rho(\mathbf{x}) \leq \lambda_1$. (5%)

   (c) Show that $\max_{\mathbf{x}\neq\mathbf{0}} \rho(\mathbf{x}) = \lambda_1$ and $\min_{\mathbf{x}\neq\mathbf{0}} \rho(\mathbf{x}) = \lambda_n$. (5%)

7. Let

$$\|\mathbf{A}\|_F = (\langle \mathbf{A}, \mathbf{A} \rangle)^{\frac{1}{2}} = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2 \right)^{\frac{1}{2}} \quad \text{for } \mathbf{A} \in \mathbb{R}^{m\times n}.$$

   The *trace* of an $n \times n$ matrix $\mathbf{C}$, denoted $tr(\mathbf{C})$, is the sum of its diagonal entries; that is,

$$tr(\mathbf{C}) = c_{11} + c_{22} + \cdots + c_{nn}$$

   If $\mathbf{A}$ and $\mathbf{B}$ are $m \times n$ matrices, show that

   (a) $\|\mathbf{A}\|_F^2 = tr(\mathbf{A}^T\mathbf{A})$　(5%)

   (b) $\|\mathbf{A}+\mathbf{B}\|_F^2 = \|\mathbf{A}\|_F^2 + 2tr(\mathbf{A}^T\mathbf{B}) + \|\mathbf{B}\|_F^2$　(5%)

8. Let $\mathbf{A}$ be a symmetric $n \times n$ matrix. Show that $e^{\mathbf{A}}$ is symmetric and positive definite. (10%)

9. Let $\mathbf{A}$ be a singular $n \times n$ matrix. Show that $\mathbf{A}^T\mathbf{A}$ is positive semidefinite, but not positive definite. (10%)