國立成功大學

113學年度碩士班招生考試試題

編　號：198

系　所：電機資訊學院-資訊聯招

科　目：計算機數學

日　期：0201

節　次：第 3 節

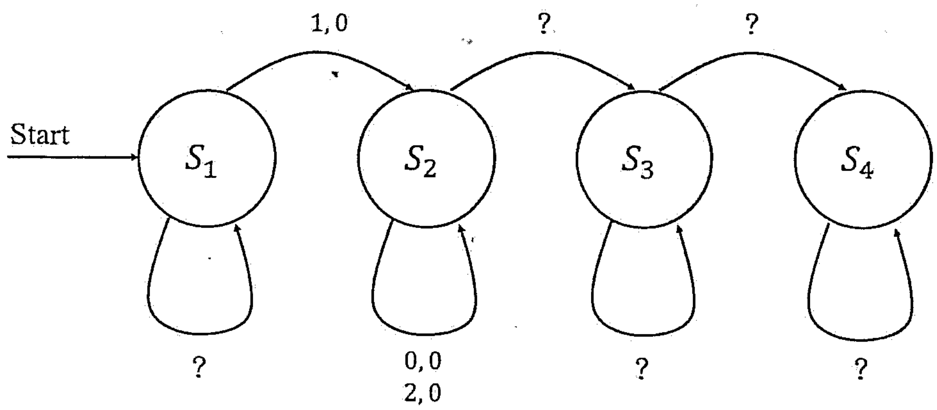
備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、離散數學 (50%)

1. (7 points) Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$ for $n \geq 0$ and $a_0 = 3000$ and $a_1 = 3300$.
2. (10 points) Please list the first 5 coefficients of the generating function $F(x) = \sqrt{1+x}$. (請化簡為最簡分數形式)
3. (15 points) If $G(x)$ is the generating function for the sequence $\{a_k\}$, what is the generating function for each of these sequences?
   (A) (5 points) $3a_0, 3a_1, 3a_2, 3a_3, \ldots$
   (B) (5 points) $0, 0, 0, 0, a_2, a_3, \ldots$
   (C) (5 points) $a_1, 2a_2, 3a_3, 4a_4, \ldots$

4. (13 points) Design a finite state machine $M = (S, \varphi, \sigma, v, \omega)$, where $S = \{s_1, s_2, s_3, s_4\}$, $\varphi = \{0,1,2\}$, $\sigma = \{0, 1\}$. The machine outputs 1 if the input string contains at least three 1s, otherwise it outputs 0. (請填問號應有的內容)



5. (5 points) Consider the trees with vertices {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} that have corresponding degrees (1, 3, 1, 3, 2, 1, 1, 3, 1, 3). How many different spanning trees are there in total?

二、線性代數 (50%)

6. (10%) Let a vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4$. It has 24 rearrangements like (x_1, x_2, x_3, x_4) and (x_4, x_3, x_1, x_2). Those 24 vectors span a subspace S. Find specific vectors $\boldsymbol{x}$ so that the dimension of S is three.

7. (10%) Let G_2 G_3 and G_4 be the determinants of the matrices in the following form:

$$G_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix},\ G_3 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix},\ G_4 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

Calculate the value of G_n.

8. (10%) Let A and B as two matrices. If B is invertible, prove that AB has the same eigenvalues as BA.

9. (10%) Consider the points $P(3,-1,4)$ and $Q(6,0,2)$, and $R(5,1,1)$. Find the point S in $\mathbb{R}^3$ whose first component is -1 and such that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RS}$.

10. True or False

(a) (2%) Every positive definite matrix is invertible.

(b) (2%) The determinant of $A - B$ equals $\det A - \det B$.

(c) (2%) If $\boldsymbol{u}$ is orthogonal to every vector of a subspace W, then $\boldsymbol{u} = \boldsymbol{0}$.

(d) (2%) If A is square and $A\boldsymbol{x} = \boldsymbol{b}$ is inconsistent for some vector $\boldsymbol{b}$, then the nullity of A is zero.

(e) (2%) If there is a basis for $\mathbb{R}^n$ consisting of eigenvectors of an $n \times n$ matrix A, then A is diagonalizable.