國立成功大學

114學年度碩士班招生考試試題

編　號：149

系　所：電機資訊學院-資訊聯招

科　目：計算機數學

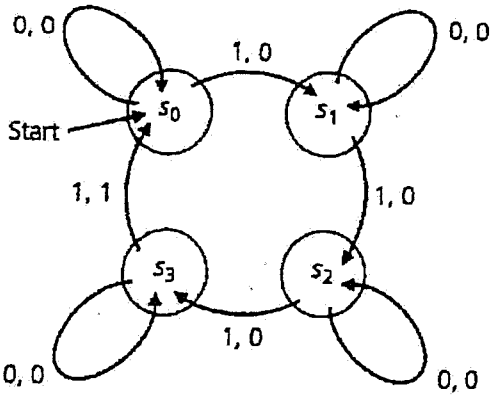
日　期：0210

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

一、離散數學 (50%)

1. Let M be a Finite State Machine shown below:



I. (10%) How many distinct input string x are there such that $||x||=8$ and $v(s_0, x)=s_0$? Please chose one answer:
(a) 72. (b) 73. (c) 74. (d) 75.

II. (10%) How many distinct input string x are there such that $||x||=12$ and $v(s_0, x)=s_0$? Please chose one answer:

(a) 991. (b) 992. (c) 993. (d) 994.

2. (10%) Find the number of solutions of $x_1 + 2x_2 + x_3 = 17$, where x_i's are nonnegative integers with $2 \le x_1 \le 5$, $3 \le x_2 \le 6$, $4 \le x_3 \le 7$.
(a) 8. (b) 9. (c) 10. (d) 11.

3. (10%) Let S = {1, 2, 3, 4, 5, {6, 7}}. Which one is the number of the power set of S? (a) 128. (b) 256. (c) 64. (d) 32.

4. (10%) Which one is the solution of the recurrence relation

$a_n - 3a_n - 1 = 5(3^n)$, where $n \ge 1$ and $a_0 = 2$?

(a) $(1+3n)(3^n)$. (b) $(2+4n)(4^n)$. (c) $(2+6n)(5^n)$. (d) $(2+5n)(3^n)$.

二、線性代數 (50%)

5. (10%) Let A be an $m \times n$ matrix with the rank r, which is the relationship between r, m, n if $A\boldsymbol{x} = \boldsymbol{b}$ has exactly one solution for some $\boldsymbol{b}$, no solution for other $\boldsymbol{b}$?
   (A) $r = m, \quad r < n$
   (B) $r < m, \quad$ always $r \leq n$
   (C) $r < m, \quad r = n$
   (D) $r = m = n$

6. (10%) Consider the matrix following matrix. Which condition must the scalars x_1, x_2, x_3 satisfy for the matrix to be nonsingular?

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

   (A) No two of the x_i values are zero.
   (B) All x_i values are distinct.
   (C) No x_i can be zero.
   (D) No x_i can be a scalar multiple of the other x_j.

7. (10%) Which one of the following is false?
   (A) If A is an $m \times n$ matrix, then AA^T and A^TA have the same rank.
   (B) If Q is an orthogonal matrix, then Q^T also is an orthogonal matrix.
   (C) If $L : V \to V$ is a linear transformation and $L(v_1) = L(v_2)$, then $v_1 - v_2$ is in the kernel of L.
   (D) If A and B are row equivalent matrices, then their determinants are equal.

8. (10%) About the eigenvalues and the corresponding eigenspaces for the following matrix, which is wrong?

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (A) One of the eigenvalues is 1.
   (B) The eigenspace corresponding to $\lambda = 5$ is (0, 0, 1, 0).
   (C) The largest eigenvalue is 5.
   (D) There are multiple identical eigenvalues.

9. True or False

(a) (2%) If a 3×2 matrix Q has orthonormal columns, then $\|Q\boldsymbol{x}\|$ always equals $\|\boldsymbol{x}\|$.

(b) (2%) Let $P = A(A^TA)^{-1}A^T$, where A is an $m \times n$ matrix of rank n. Then $P^k = P$ for $k = 1,2,\ldots$.

(c) (2%) If $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k\}$ is an orthonormal set of vectors in $\mathbb{R}^n$ and $U = (\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_k)$, then $UU^T = I_n$ (the $n \times n$ identity matrix).

(d) (2%) If A is symmetric and $\det(A) > 0$, then A is positive definite.

(e) (2%) If A is symmetric positive definite, then the diagonal elements of A must all be positive.