

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Prove that  $\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A}^T)$  for every  $m \times n$  matrix. (20%)
2. Derive the integration formula (20%)

$$\int_0^{\infty} \frac{\cos(ax) - \cos(bx)}{x^2} dx = \frac{\pi}{2}(b - a) \quad (a \geq 0, b \geq 0)$$

Then, with the aid of the trigonometric identity  $1 - \cos(2x) = 2\sin^2x$ , point out how it follows that

$$\int_0^{\infty} \frac{\sin^2x}{x^2} dx = \frac{\pi}{2}$$

3. Use the method of separation of variables to solve the problem: (20%)

$$u_t = 9u_{xx} \quad 0 < x < 8, t > 0$$

$$u(0, t) = u(8, t) = 0, \quad t > 0$$

$$u(x, 0) = x + 2, \quad 0 < x < 8$$

4. Solve the following initial value problem: (20%)

$$y'' + 4y' + 8y = e^{2t} - 2\delta(t - 2\pi), \quad y(0) = 2, \quad y'(0) = 0$$

5. Let  $\bar{R}(s)$  be a space curve represented by the arc length parameter  $s$ . (20%)

(a) What is the unit tangent vector  $\bar{T}(s)$  along the space curve? (5%)

(b) Represent the unit normal vector  $\bar{N}(s)$  along the space curve by use of  $\bar{T}(s)$  and the curvature  $\kappa$  of the curve. (5%)

(c) Let  $f(x, y, z) = x^2 + y^2 - z$  be a temperature field. What is the rate of change of  $f(x, y, z)$  at the point  $P(1, 1, 2)$  on the space curve  $\bar{R}(s)$  in the direction of the vector  $\bar{v} = 2\bar{i} + 2\bar{j} + \bar{k}$ . (5%)

(d) In problem (c), what is the maximum value of the rate of change of  $f(x, y, z)$  at the point  $P(1, 1, 2)$ ? (5%)