

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Evaluate the integral

$$\iint_R \frac{1}{\sqrt{(x-y)^2 + 2(x+y) + 1}} dA,$$

where R is the region bounded by the graph of $y = x$, $x = 2$, and $y = 0$, by means of the change of variables $x = u + uv$, $y = v + uv$.

2. (20%) Find the orthogonal matrix P and the diagonal matrix L that

$$P^T \begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix} P = L \text{ and } |\det P| = 1.$$

3. (20%)

A differential equation is given by

$$L[y] = 3x^2 - \sin 2x \quad (1)$$

where $L[y]$ is a linear operator and is defined by

$$L[y] = y'' + 4y \quad (2)$$

a) First find the homogeneous solution y_h , that is

$$L[y_h] = 0 \quad (3) \quad (6 \text{ 分})$$

b) Verify that your solution set of y_h is linearly independent (6 分)

c) Find the general solution of Equation (1) (8 分)

4. (20%) Consider the problem

$$u_{xx} - u_t = 0, \quad 0 < x < 2, t > 0$$

$$u_x(0, t) = 0, u(1, t) = 100, \quad t > 0$$

$$u(x, 0) = f(x), \quad 0 < x < 2$$

where f is the piecewise constant function

$$f(x) = \begin{cases} 60, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$

Please use the method of separation of variables to find the solution.

5. (20%) Using the residue theorem, evaluate

$$I = \int_0^{\infty} \frac{\cos(kx)}{1+x^2} dx$$

with $k < 0$.