

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Compute the following line integral:

$$\int_C xdy + y^2 dx$$

C is the curve: $x = t^2, y = 2t, 0 < t < 2$

2. (20%) Compute the maximum value for the following function

$$f(x, y, z) = x + 4y + 4z$$

Under the constraint: $x^2 + y^2 + z^2 = 1$

3. (20%) Evaluate the integrals

(a) (10%)

$$\int_0^{\pi} x \cos(x) dx$$

(b) (10%)

$$\int_1^2 \int_0^y \sin(x-y) dx dy$$

4. (20%) Find the orthogonal matrix P and the diagonal matrix L that

$$P^T \begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix} P = L \text{ and } |\det P| = 1.$$

5. (20%)

a) (10%) First find two linear independent homogeneous solutions:

$$y'' + 4y = 0$$

b) (10%) Find the general solutions of the following ODE

$$y'' + 4y = 3x^2 - \sin 2x$$