

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Problem 1: (20%)

Find the rank, the nullity, the row space, and the null space of the matrix shown below:

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 2 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & -1 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$$

Problem 2: (20%)

Derive $y(t)$ for the ordinary differential equation given in the following:

$$\ddot{y} + 3\dot{y} + 2y = f(t), \text{ where } y(0) = 2 \text{ and } \dot{y}(0) = -1, \text{ and } f(t) = \begin{cases} 0, & 0 \leq t < 1, \\ 1, & 1 \leq t < \infty. \end{cases}$$

Problem 3: (20%)

(a). $f(x, y, z) = x+y+z$, $S: z = x+y, 0 \leq y \leq x, 0 \leq x \leq 1$

Find $\iint_S f(x, y, z) dA = ?$

(b). $\vec{F} = z^2\vec{i} + 4x\vec{j}$, $S: z = 1, 0 \leq y \leq 1, 0 \leq x \leq 1$

Find $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dA = ?$

Problem 4: (20%)

(a). Find $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 1} dx$

(b). Find $\int_{-\infty}^{\infty} \frac{\cos 4x}{(x^2 + 1)(x^2 + 4)} dx$

Problem 5: (20%)

Show that the following partial differential equation:

$$u_t = u_{xx} + 6u_x + 8u, \quad 0 < x < 2, t > 0$$

$$u(0, t) = u(2, t) = 0, \quad t \geq 0$$

$$u(x, 0) = x(2 - x), \quad 0 \leq x \leq 2$$

can be transformed into a standard heat equation by choosing appropriately α, β

and letting $u(x, t) = e^{\alpha x + \beta t} v(x, t)$

Please find the values of α, β and the partial differential equation for $v(x, t)$ and its relative boundary and initial conditions.