

國立成功大學

110學年度碩士班招生考試試題

編 號： 134、138、166
系 所： 航空太空工程學系
 民航研究所
 能源工程國際碩士學位學程
科 目： 工程數學
日 期： 0202
節 次： 第 3 節
備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. $\mathbf{F} = 3x^2y^2\mathbf{i} + y\mathbf{j} - 6zxy^2\mathbf{k}$; D the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$. Please use the divergence theorem to find the outward flux $\iint_S (\mathbf{F} \cdot \mathbf{n})dS$ of the given vector field \mathbf{F} . (15%)

2. (a) Please use Fourier Transform to solve this:

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, -\infty < x < \infty, t \geq 0$$

$$u(x, 0) = f(x), \frac{\partial u}{\partial t}(t = 0) = g(x), -\infty < x < \infty \quad (10\%)$$

(b) If $g(x)=0$, show that the solution of part (a) can be written

$$\text{as } u(x, t) = \frac{1}{2}[f(x + at) + f(x - at)]. \quad (5\%)$$

3. (a) Consider a complex-valued function:

$$f(z) = \frac{-2z + 3}{z^2 - 3z + 2}$$

with a center 0. Find all Taylor and Laurent series. (10%)

(b) Evaluate the following integral counterclockwise around a closed path C .

$$\oint_C \frac{4 - 3z}{z^2 - z} dz$$

where $C: |z| = 2$. (10%)

4. Solve the given initial-value problem.

(a) $y' + (\tan x)y = \cos^2 x, \quad y(0) = -1. \quad (5\%)$

(b) $4x^2y'' + 17y = 0, \quad y(1) = -1, \quad y'(1) = -\frac{1}{2}. \quad (10\%)$

5. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, $0 \leq x \leq 1$, $t \geq 0$ (15%)

$$u(0, t) = \cos t, \quad u(1, t) = 0, \quad t \geq 0$$

$$u(x, 0) = 0, \quad 0 \leq x \leq 1.$$

6. (a) Solve the initial value problem (15%)

$$\frac{d^2 x}{dt^2} + 2t \frac{dx}{dt} - 4x = 1$$

$$\text{with initial conditions } x(0) = x'(0) = 0$$

(b) For a system $Y(S) = G(S)R(S)$, $r(t)$ is a unit step input and

$$G(S) = \frac{S + 2}{S^3 - 8S^2 + 3S + 2}$$

Determine the steady state value of $y(t)$. (5%)