

國立成功大學

113學年度碩士班招生考試試題

編 號： 133、137、161、164

航空太空工程學系

系 所： 民航研究所

能源工程國際碩士學位學程

太空系統工程研究所

科 目： 工程數學

日 期： 0201

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Evaluate: $\oint_c \left(\frac{ze^{\pi z}}{z^4 - 16} + ze^{\pi/z} \right) dz$ $c: 9x^2 + y^2 = 9$ (counterclockwise)

2. (20%) Solve the following differential equation, showing the details of your work:

(a) $y'' - 4y' - 12y = \sin(2t) + te^{4t}$

(b) $y''' - 12y'' + 48y' - 64y = 12 - 32e^{-8t} + 2e^{4t}$

3. (20%) Attempt to solve the heat equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T(r, T)}{\partial t} \quad \text{in } a \leq r < b, \quad t > 0$$

BC1: $T(a, t) = 0$; BC2: $T(b, t) = 0$

IC: $T(r, 0) = F(r)$

4. (20%) Use the differential equation: $\frac{d^2 \dot{x}}{dt^2} + a^2 x = 0 \quad t \geq 0$

with appropriate initial conditions to show that: $\mathcal{L}(\cos at) = \frac{s}{s^2 + a^2}$ and $\mathcal{L}(\sin at) = \frac{a}{s^2 + a^2}$

(you may not use integration in this question)

5. (20%) The function $\varphi = \varphi(x, y)$ satisfies Laplace's equation in Cartesian coordinates,

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \quad \text{in the part of the } x\text{-}y \text{ plane for which } y \geq 0.$$

It is further given that:

- $\varphi(x, y) \rightarrow 0$ as $\sqrt{x^2 + y^2} \rightarrow 0$

- $\varphi(x, 0) = \begin{cases} 1/2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$

Use Fourier transforms to show that $\varphi(x, y) = \frac{1}{\pi} \int_0^\infty \frac{1}{k} (e^{-ky} \sin k \cos kx) dk$

and hence deduce the value of $\varphi(\pm 1, 0)$