1. (a) Show that the differential form
$$2\sin 2x \sinh y \, dx - \cos 2x \cosh y \, dy$$
is exact. (5%)

(b) Solve the differential equation (5%)
$$2\sin 2x \sinh y \, dx - \cos 2x \cosh y \, dy = 0.$$

(c) Solve the initial value problem. (10%)
$$y'' - y = 2\cos x, \; y(0) = 0, \; y'(0) = 3$$

2. (a) Calculate the integral (10%)
$$\int_C \vec{F}(\vec{r}) \cdot d\vec{r}, \; \vec{F} = [2z, \, 1, \, -y], \; C : \vec{r} = [\cos t, \, \sin t, \, 2t] \; from$$
$$(0, \, 0, \, 0) \; to \; (1, \, 0, \, 4\pi).$$

(b) Evaluate the integral (10%)
$$\int_{(\pi/2, \, -\pi)}^{(\pi/4, \, 0)} (\cos x \cos 2y \, dx - 2\sin x \sin 2y \, dy).$$

3. Solve $y''' - y' = \sin t$ by Laplace transform, given $y(0) = 2$, $y'(0) = 0$, and $y''(0) = 1$. Note that $L\left\{\dfrac{\sin \omega t}{\omega}\right\} = \dfrac{1}{s^2 + \omega^2}$ and $L\{\cos \omega t\} = \dfrac{s}{s^2 + \omega^2}$.

(20pt)

（背面仍有題目，請繼續作答）

4. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$.

(20 pt)

　　(a). The determinant $\det(-A^{10}) = ?$

　　(b). The eigenvalues $\lambda(A^{10}) = ?$

　　(c). The rank $rank(A) = ?$

　　(d). How many linearly independent eigenvectors does matrix A have ?

5. Let $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$. Assume that C is the trace (邊緣) of the cylinder $x^2 + y^2 = 1$ in the plane

(20 pt) $y + z = 2$. Orient C counterclockwise as viewed as from above. See the following figure.

　　(a). Calculate the surface integral $I_1 = \iint_S (\nabla \times \vec{F}) \bullet \vec{n}\, dS$ directly, where $\vec{n}$ is the outward unit

　　　　normal of the surface S enclosed by C, and dS is the differential area element of the surface S.
　　　　(Note that you are prohibited to evaluate I_1 by using the result in (b).)

　　(b). Calculate the line integral $I_2 = \oint_C \vec{F} \bullet d\vec{R}$ directly.

　　　　where $d\vec{R} = dx\vec{i} + dy\vec{j} + dz\vec{k}$ is the differential displacement along C.
　　　　(Note that you are prohibited to evaluate I_2 by using the result in (a).)