

本試題是否可以使用計算機： 可使用， 不可使用（請命題老師勾選）

20%[1]

(a). Find the general solution to the following equation:  $\frac{d^2 y}{dt^2} - \frac{dy}{dt} - 2y = 3e^{2t}$ .

(b). Find the solution to the above equation for  $y(0) = 3$  and  $\left. \frac{dy}{dt} \right|_{t=0} = 4$ .

[2]

Solve the system of equations  $\dot{x}(t) = Ax(t) + B(t)$ ,

where  $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ ,  $B(t) = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ ,  $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ ,  $x(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

5%(a). What are the eigenvalues of matrix  $A$ ?

15%(b). What is the solution  $x(t)$  to the system?

20%[3]

(a). Find the coordinates  $(x, y, z)$  of the point on the plane

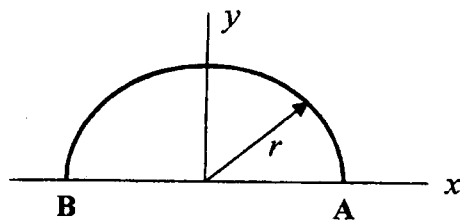
$$ax + by + cz = d \quad \text{with} \quad a^2 + b^2 + c^2 = 1,$$

which is closest to the origin. Also find the shortest distance from the origin to the plane.

(b). Evaluate the line integral

$$I = \int_C f \, ds,$$

where  $f = x^2 y$  and  $C$  is the semicircular arc of radius  $r$  from  $A$  to  $B$  as shown in the figure.



(背面還有題目，請繼續作答)

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20%[4]

Use the Fourier series method to solve the problem:

$$u_t = 4u_{xx} \quad 0 < x < 2, t > 0$$

$$u(0, t) = u(2, t) = 0, \quad t > 0$$

$$u(x, 0) = 2 - 2\cos(\pi x) + 4\sin(2\pi x), \quad 0 < x < 2$$

[5]

12% (a). State the sufficient and necessary conditions such that a complex function is analytic. How about the relation between these conditions and the uniqueness of the derivative  $f'(z)$ ? Please use  $f(z) = \frac{1}{1-z}$  to explain your answer. Is this  $f(z)$  analytic at the point  $z = 1$ ? Is this  $f(z)$  analytic at points satisfy  $|z| > 0.001$ ?

8% (b). Find the complex integration  $\oint_C \frac{1}{1-z} dz = ?$  along the following closed curves in the counterclockwise sense.

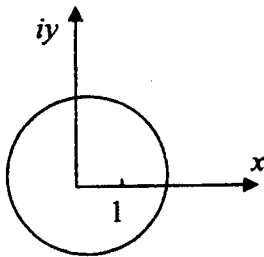


Figure 1

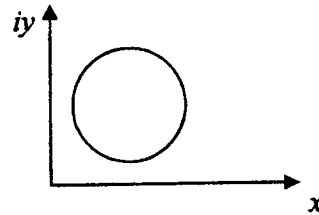


Figure 2