編號： 140、144、175　　國立成功大學 106 學年度碩士班招生考試試題

系　所：航空太空工程學系、民航研究所、能源國際碩士學位學程

考試科目：工程數學　　考試日期：0213，節次：3

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Problem 1: (20%)

Find the rank, the nullity, the row space, and the null space of the matrix shown below:

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 2 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & -1 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$$

Problem 2: (20%)

Derive y(t) for the ordinary differential equation given in the following:

$\ddot{y} + 3\dot{y} + 2y = f(t)$, where $y(0) = 2$ and $\dot{y}(0) = -1$, and $f(t) = \begin{cases} 0, & 0 \le t < 1, \\ 1, & 1 \le t < \infty. \end{cases}$

Problem 3: (20%)

(a). $f(x, y, z) = x+y+z$,　S: $z = x+y$, $0 \le y \le x$, $0 \le x \le 1$

Find $\iint_S f(x,y,z)dA = ?$

(b). $\vec{F} = z^2\vec{i} + 4x\vec{j}$,　S: $z = 1$, $0 \le y \le 1$, $0 \le x \le 1$

Find $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dA = ?$

Problem 4: (20%)

(a). Find $\int_{-\infty}^{\infty} \frac{x\sin x}{x^2+1} dx$　　(b). Find $\int_{-\infty}^{\infty} \frac{\cos 4x}{(x^2+1)(x^2+4)} dx$

Problem 5: (20%)

Show that the following partial differential equation:

$$u_t = u_{xx} + 6u_x + 8u, \qquad 0 < x < 2, t > 0$$
$$u(0,t) = u(2,t) = 0, \qquad t \ge 0$$
$$u(x,0) = x(2-x), \qquad 0 \le x \le 2$$

can be transformed into a standard heat equation by choosing appropriately α, β and letting $u(x,t) = e^{\alpha x + \beta t} v(x,t)$

Please find the values of α, β and the partial differential equation for $v(x,t)$ and its relative boundary and initial conditions.