

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. For a differential equation  $2xy' + y = 0$

- (2%) Show that  $2xy' + y = 0$  can be written as  $ydx + 2xdy = 0$
- (4%) Solve the differential equation using separable method
- (2%) Is this differential equation exact?
- (7%) If yes, solve. If not, look for an integrating factor and solve.

2. Solve the initial value problem,  $y'' - 6y' + 9y = e^{3x}$ ,

- (3%) Solve the homogeneous solution
- (5%) Solve the nonhomogeneous solution
- (2%) Solve the initial problem,  $y(0) = 1, y'(0) = 1$

3. (15%)

Using Frobenius Method to solve  $x^2y'' + (x^2 + x)y' - y = 0$ . Try to identify the series as expansions of known functions. Show the details of your work.

4. (20%)

Use Laplace inverse to solve the following equation:

$$Y(s) = \frac{1.151s + 0.1774}{s^4 + 0.739s^3 + 0.921s^2}$$

5. (20%)

$$\frac{\partial^2 T}{\partial x^2} + x = \frac{\partial T}{\partial t} \quad \text{in } 0 < x < 1, \text{ for } t > 0$$

with the boundary conditions of

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x=0, \text{ for } t > 0$$

$$T = a \quad \text{at } x = 1, \text{ for } t > 0$$

and the initial condition of  $T = a$  at  $t = 0$ , in  $0 \leq x \leq 1$

Find the solution of  $T(x, t)$ .

6. (20%)

Find the finite Fourier cosine transform of the function:

$$a) f(x) = \begin{cases} 2 & \text{for } 0 \leq x \leq 0.5 \\ -1 & \text{for } 0.5 < x \leq \pi \end{cases} \quad (10\%)$$

$$b) f(x) = e^{-x} \quad 0 \leq x \leq \pi \quad (10\%)$$