

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. 【20%】A random experiment has a sample space $\{a, b, c, d\}$. These outcomes are not equally-likely; their probabilities are: 0.1, 0.3, 0.5, 0.1. Let Event $A = \{a, b\}$, $B = \{b, c, d\}$, and $C = \{d\}$. A' is the complement of event A. Determine the following probabilities: (a) $P(A)$; (b) $P(B)$; (c) $P(A')$; (d) $P(A \cap B)$; (e) $P(A \cup B)$
2. 【20%】 For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution (the probability density function is $f(x) = e^{-\lambda} \lambda^x / x!$) with a mean of 2.3 flaws per mm. (a) Find the probability of exactly 2 flaws in 1 mm of wire; (b) Determine the probability of 10 flaws in 5 mm of wire; (c) Determine the probability of at least 1 flaw in 2 mm of wire.
3. 【20%】 Suppose that the current measurements in a strip of wire are assumed to follow a **Normal** distribution with $\mu = 10$ and $\sigma = 2$ mA. (a) What is the probability that the current measurement is between 9 and 11 mA? (b) Determine the value for which the probability that a current measurement is below 0.98. [Hint: Assume Z is a standard normal random variable. $P(Z \leq -0.5) = 0.30854$; $P(Z \leq 0.5) = 0.69146$; $P(Z \leq 1.5) = 0.93319$; $P(Z \leq 2.05) = 0.98000$]
4. 【20%】 Use the least-square method to derive the slope of a simple linear regression model $y = \beta_0 + \beta_1 x$ is $\beta_1 = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$ where (x_i, y_i) is the n paired data. \bar{x} and \bar{y} are the mean values of x and y , and write the definition of coefficient of determination.
5. 【20%】 Us the Method of Moment to derive the parameters of Gamma distribution. [Hint: PDF of Gamma distribution $f(x) = \lambda^r x^{r-1} e^{-\lambda x} / \Gamma(r)$ and the first and second moments of Gamma distribution are r / λ and $r(r+1) / \lambda^2$ respectively.]