

1. **Discuss** the effect of the following phenomena on the unbiasedness and the variance of the OLS estimator. (20%) [Your score depends on how completeness your answer is.]
- Heteroskedasticity
 - Including an irrelevant variable
 - Omitting a relevant variable.
 - Multicollinearity

2. Suppose that you want to estimate the following relation:

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \quad (\text{Model 1})$$

Note: All the classical assumptions are satisfied for the model 1 above.

Suppose now that unfortunately Y_i is not observable, you can only observe

$$Y_i^* = Y_i + \mu + v_i \quad (\text{Model 2})$$

Where μ is an unknown constant parameter, v_i is random with $E(v_i) = 0$, $E(v_i^2) = \text{Var}(v_i)$ for all i , and $E(v_i v_j) = 0$ for all $i \neq j$, $E(v_i \varepsilon_j) = 0$ for all i and j .

- 2a) Rearrange Model 1 in order to express it on form of the observable variables (Y_i^*, X_i) :

$$Y_i^* = \gamma_1 + \gamma_2 X_i + u_i \quad (\text{Model 3})$$

Stating clearly the relation between the elements of Models 1 and Model 2. (7%)

- 2b) Is $\hat{\gamma}_2$, the OLS estimator of the coefficient of X_i in model 3, an unbiased estimator of β_2 (the coefficient of X_i in model 1)? Please show your answer. (7%)

- 2c) Is $\hat{\gamma}_1$, the OLS estimator of the intercept in model 3, an unbiased estimator of β_1 (the intercept in model 1)? Please show your answer. (7%)

3. Please find the MGF of the Poisson distribution, and use the MGFs to find the variance of the Poisson distribution. (12%)

(背面仍有題目,請繼續作答)

系所組別： 經濟學系

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4. The random variable X has a continuous distribution $f(x)$ and cumulative distribution function $F(x)$. What is the probability distribution of the sample maximum? (6%)
5. Using the following joint probability distribution (16%)

		X		
		0	1	2
Y	0	0.05	0.10	0.03
	1	0.21	0.11	0.19
	2	0.08	0.15	0.08

- 5a) Please find $Var(Y)$
- 5b) Please find $Cov(X, Y)$
- 5c) Please find $E(X^2 Y^3)$
- 5d) Please find $f_x(X = 2 | Y > 0)$

6. Consider a linear model to explain monthly beer consumption:

$$beer = \beta_0 + \beta_1 income + \beta_2 price + \beta_3 education + \beta_4 female + u$$

$$E(u | income, price, education, female) = 0$$

$$Var(u | income, price, education, female) = \sigma^2 income^2$$

Please describe the estimation steps for getting the unbiased and efficient estimators. (10%)

7. The following equation explains weekly hours of television viewing by a child in terms of the child's age, mother's education, father's education and number of siblings.

$$TV^* = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 mothereducation + \beta_4 fathereducation + \beta_5 sibling + u$$

We are worried that TV^* is measured with error in our survey. Let TV denote the reported hours of television viewing. That is, $TV = TV^* + e_0$, where e_0 is measurement error. Under what condition, does OLS estimation produce unbiased estimators for $\beta_1 \sim \beta_5$. Explain. (15%).