1. (20%) Assume John purchases goods 1 and 2 with the amounts q_1 and q_2 respectively. His utility function is $u(q_1, q_2) = \sqrt{q_1} + \frac{1}{2}q_2$.
(a.) (10%) Find his expenditure to achieve utility level $\bar{u}$ for prices (p_1, p_2).
(b.) (10%) Assume his income is 11 and the initial prices are $(1, 1)$. If the price of good 1 rises to 2, please find his equivalent variation.

2. (15%) Assume John spends his one-day earning to buy x units of some good with price 1. His utility function is $u = x^a (24 - h)^b$, where h is his work hours in one day. Assume the wage is w per hour. Find his labor supply function.

3. (15%) Suppose a firm hires L hours of labor services and rents K hours of machine services. Assume that the firm's production function $f(L, K)$ exhibits constant returns to scale and the wage of labor and the rental rate of machine are constant in the long run. Find the average cost function of the firm.

4. (25%) Suppose a competitive firm can produce $q = 4L^{1/4}K^{1/2}$ units of outputs each month when hiring L hours of labors to work with K machines. Let p be the price for its products, w be the hourly wage rate, and r be the monthly rent for each machine. Suppose the firm is also a competitive buyer in both labor and capital markets.
(a.) (5%) Derive $L^D_{SR}(w, K_0)$, its short run labor demand function when working with K_0 machines.
(b.) (10%) Derive its long run labor demand function, $L^D_{LR}(w)$.
(c.) (5%) Suppose at w_0, $L^D_{LR}(w_0) = L^D_{SR}(w_0, K_0)$. Show that for $w > w_0$, $L^D_{LR}(w) < L^D_{SR}(w, K_0)$.
(d.) (5%) Mathematically show that $L^D_{LR}(w)$ is more elastic than $L^D_{SR}(w, K_0)$ at any w.

5. (25%) A firm has invented a new product. The estimated demand for its new product is $q = 100 - 2p$ when the price is set at p. Suppose the product is patented so that the firm can act as a monopolist. Suppose there's no any production cost.
(a.) (5%) Find the optimal price the firm should set in order to maximize its profit.
(b.) (15%) Suppose the firm can boost its demand with TV commercials. If x is spent on the advertisement, the demand will shift up to $q = 100 - 2p + (2 - \frac{x}{p})x$. Suppose the firm will first determine the amount of spending on commercials, x, and then the price for its products, p. Find the optimal x, the amount the firm should spend on TV commercials.
(c.) (5%) Continue on (b.). However, suppose now the firm first has to pay additional \$1,000 to make the film to be used in TV commercials besides its advertising spending, x, unless the firm decides not to spend any money on advertisements. Find again the optimal x in this case.