※ 考生請注意：本試題不可使用計算機 請勿在本試題紙上作答，否則不予計分

Please read the instruction carefully before answering the questions: The exam consists of 25 multiple-choice questions. All questions are equally weighted and there is a single best answer for each question. The exam is worth 100 points. Please number the question you are answering; otherwise, no points will be given. The distribution table is attached to the end of the exam for your reference.

Multiple-Choice Questions: (100 points, 4 points each)

Identify One Best Answer for Each Question

Questions 1—4 all concern the following situation. Suppose we have three data sets (Set 1, Set 2 and Set 3), each with sample size, $n = 100$. The data points in Set 1 are all equal to 5. In Set 2, the first 99 data points are equal to 5, and the last value is 6. In Set 3, the first 99 data points are equal to 5, and the last value is 4.

1. Which data set has the largest sample mean?

   (A) Set 1
   (B) Set 2
   (C) Set 3
   (D) They are all the same
   (E) It cannot be determined.

2. Which data set has the largest sample median?

   (A) Set 1
   (B) Set 2
   (C) Set 3
   (D) They are all the same
   (E) None of the above.

3. Which data set has the smallest sample standard deviation?

   (A) Set 1
   (B) Set 2
   (C) Set 3
   (D) They are all the same
   (E) It cannot be determined.

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4. Which data set has the largest interquartile range?

   (A) Set 1
   (B) Set 2
   (C) Set 3
   (D) They are all the same
   (E) It cannot be determined.

5. In a data set with sample size $n = 2$, the range is 10 and the sample mean is 5. Then the sample variance is:

   (A) 10
   (B) 25
   (C) 50
   (D) 2
   (E) None of the above.

6. Suppose A and B are events with $P(A) \neq 0$, $P(B) \neq 0$, and $P(B|A) = P(A|B)$. Then it must be true that:

   (A) Events A, B are independent.
   (B) $P(A) + P(B) = 1$.
   (C) $P(A) = P(B)$ or A, B are mutually exclusive.
   (D) $P(A) = 1 - P(B)$.
   (E) None of the above.

7. A fair coin is tossed three times, independently. Let A={At least two heads}, B={Heads on first toss}. Then A, B are

   (A) Independent
   (B) Mutually Exclusive
   (C) Dependent
   (D) Events of zero probability
   (E) None of the above.

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8. A single card is drawn at random from a freshly-shuffled deck. If A={Card is Red} and B={Card is a Diamond} then $P(A \cap B)$ is:

   (A) .5
   (B) .75
   (C) .25
   (D) .125
   (E) None of the above.

9. An economist says that the Great Recession is twice as likely to continue in 2013 as it is to end in 2013. Then, in the opinion of the economist, the odds (yielding a fair bet) that the Great Recession will end in 2013 are:

   (A) 1:1
   (B) 2:1
   (C) 1:2
   (D) 6:5
   (E) None of the above.

10. A prospective MBA student would like to get a score greater than 700 on the GMAT exam. The applicant plans to take the exam several times. Assume that the results on the successive tests are independent, and that for each test the applicant has an 80% chance of getting a score exceeding 700. How many GMAT exams should the applicant plan to take in order to have a 99.2% chance that at least one of the resulting scores will exceed 700?

    (A) 1
    (B) 2
    (C) 3
    (D) 4
    (E) None of the above.

11. Consider a game where you win \$9 with probability .4 and lose \$6 with probability .6. What is the standard deviation of your profit for this game?

    (A) 7.348
    (B) 54
    (C) 2.149
    (D) 117
    (E) None of the above.

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12. Suppose that the number of cars manufactured per day in an auto factory has a mean of 75 and a standard deviation of 12. If the average number of cars manufactured over the next 36 days exceeds 77, then each worker will receive a $1000 bonus. What is the expected value of the bonus for each worker?

(A) $159

(B) $433

(C) $841

(D) $568

(E) None of the above.

13. A fast food restaurant is running a promotion. To get the promotional price, customers need to order one small soda and two sandwiches (not necessarily different sandwiches). The restaurant has three types of soda and eight types of sandwiches. Overall, how many different orders can the customer place?

(A) 108

(B) 84

(C) 168

(D) 192

(E) None of the above.

14. Suppose that X is a binomial random variable, and that the mean of X is twice the variance of X. Furthermore, suppose that $Prob\{X = 1\} = 1/32$. Then what is $Prob\{X = 2\}$?

(A) .3125

(B) .1094

(C) .2188

(D) It cannot be determined from the given information

(E) None of the above.

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15. Consider two independent samples from the same population. Sample 1 is of size $n = 5$, while sample 2 is of size $n = 10$. The two corresponding sample means are $\bar{x}_1$ and $\bar{x}_2$. Which of the following is true:

(A) $\bar{x}_1$ must be closer to the population mean than $\bar{x}_2$.

(B) $\bar{x}_2$ must be closer to the population mean than $\bar{x}_1$.

(C) $\bar{x}_1$ is more likely than $\bar{x}_2$ to be within one unit of the population mean.

(D) $\bar{x}_2$ is more likely than $\bar{x}_1$ to be within one unit of the population mean.

(E) None of the above statements will always be correct.

16. Here is a new way to do a hypothesis test of $H_0 : \mu = \mu_0$ versus $H_A : \mu \neq \mu_0$ based on a sample of size 200. First, split data set into two halves (100 data points each), selected at random. For each of the two halves, construct the p-value appropriate for this test, in the usual way. Then, reject H_0 if either (or both) of the two p-values is less than .05. What is the actual significance level of this test?

(A) .10

(B) .025

(C) .05

(D) .0975

(E) None of the above.

17. Suppose we wish to test $H_0 : \mu = \mu_0$ versus $H_A : \mu > \mu_0$ based on a large sample. If H_A is true, then the probability that the p-value is less than .05 is:

(A) .05

(B) Greater than .05

(C) Less than .05

(D) It cannot be determined.

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18. Suppose the amount of time a person spends browsing the NCKU website has an expected value of 5 minutes and a standard deviation of 2 minutes. If 100 people browse the NCKU website on a certain day, what is the expected value of the average amount of time that they will spend browsing the website?

    (A) 2 minutes

    (B) .02 minutes

    (C) 5 minutes

    (D) .05 minutes

    (E) None of the above.

19. Suppose scores on a test are normally distributed with a mean of 70 and a standard deviation of 10. Below what score should 69.1% of all scores lie?

    (A) 65

    (B) 75

    (C) 70

    (D) 0

    (E) None of the above.

20. For a given data set, if the 95% confidence interval for μ is $(.004, .396)$ and the 99% confidence interval for μ is $(-.0576, .4576)$, then at which significance level(s) can the null hypothesis $H_0 : \mu = 0$ be rejected in favor of the alternative hypothesis $H_A : \mu \neq 0$?

    (A) .01 but not .05

    (B) .05 and .01

    (C) .05 but not .01

    (D) Neither .05 nor .01

    (E) None of the above.

21. Based on Question 20, in the 95% confidence interval for μ, what is the probability that μ falls in the interval of $(.004, .396)$?

    (A) 5%

    (B) 95%

    (C) 99%

    (D) 100%

    (E) None of the above.

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22. If the assumptions for the simple linear regression model are all satisfied and the sample size is $n = 6$, then what is the probability that half of the data points (that is, exactly three of the points) will lie above the true regression line?

(A) 1/2
(B) .3125
(C) .015625
(D) .14286
(E) None of the above.

23. The sample mean, or arithmetic mean, of a sample data must be:

(A) a measure of center of the sample.
(B) a measure of central tendency of the sample.
(C) a positive number.
(D) Both (A) and (B) are true.
(E) None of the above.

24. X and Y are random variables. The mean of X is 0, the variance of X is 4 and $\boldsymbol{E}(Y|X) = 3 - 5X$. Which of the following is true?

(A) $\boldsymbol{E}(XY) = -20$
(B) $\text{Cov}(X, Y) = -20$
(C) X and Y are independent.
(D) Both (A) and (B) are true.
(E) None of the above.

25. Suppose we have $n = 10$ observations generated by the simple linear regression model, with $\sigma = 2$. What is the expected value of the number times the error exceeds 1.2?

(A) 5.49
(B) 2.74
(C) 1.15
(D) 2.30
(E) None of the above.

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Statistical Table:

The Normal Distribution

$$\Pr[X \le x] = \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dy$$

$$\Phi(-x) = 1 - \Phi(x)$$

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0.00	0.500	1.10	0.864	2.05	0.980
0.05	0.520	1.15	0.875	2.10	0.982
0.10	0.540	1.20	0.885	2.15	0.984
0.15	0.560	1.25	0.894	2.20	0.986
0.20	0.579	1.282	0.900	2.25	0.988
0.25	0.599	1.30	0.903	2.30	0.989
0.30	0.618	1.35	0.911	2.326	0.990
0.35	0.637	1.40	0.919	2.35	0.991
0.40	0.655	1.45	0.926	2.40	0.992
0.45	0.674	1.50	0.933	2.45	0.993
0.50	0.691	1.55	0.939	2.50	0.994
0.55	0.709	1.60	0.945	2.55	0.995
0.60	0.726	1.645	0.950	2.576	0.995
0.65	0.742	1.65	0.951	2.60	0.995
0.70	0.758	1.70	0.955	2.65	0.996
0.75	0.773	1.75	0.960	2.70	0.997
0.80	0.788	1.80	0.964	2.75	0.997
0.85	0.802	1.85	0.968	2.80	0.997
0.90	0.816	1.90	0.971	2.85	0.998
0.95	0.829	1.95	0.974	2.90	0.998
1.00	0.841	1.96	0.975	2.95	0.998
1.05	0.853	2.00	0.977	3.00	0.999