※ 考生請注意：本試題不可使用計算機

1. (15%) A company is evaluating the number of plants it shall build in order to manufacture its patented products. In addition to the annual maintenance cost of $10,000$ per plant, each new factory bears the same total costs, $c(q_i) = 50q_i + 4q_i^2$, when it produces q_i units.

(a.) (5%) If the firm will produce 400 units of products in total, how many new plants shall be built in order to produce in the most efficient way?

(b.) (10%) If the firm expects an annual inverse demand for its products as $P = 1000 - 0.5Q$ when Q units are sold, find the optimal number of plants it will build.

2. (25%) Suppose two local coffee manufacturers are the only places villagers in a remote island can supply their labors. Their collective inverse supply function is $w = 900 + L$ where w is the minimum hourly wage rate to induce a combined L hours of labor supply. With l hours of labors, Firm 1, located near down town, can produce $2l$ pounds and Firm 2 can produce $3l$ pounds of coffee respectively. Though Firm 2 is more productive, it resides on a rural area. In order to hire workers, Firm 2 compensates its workers another $300 hourly for their transportation costs. Assume these villagers eventually receive the same wage rate either working in Firm 1 or 2 so they feel indifferent working in either place. Moreover, the coffee products of each firm are shipped directly to the international market which is considered under perfect competition.

(a.) (10%) Suppose the price of coffee in the international market is $1,200 per pound, find the hours of labors hired by each firm.

(b.) (5%) Suppose the price of coffee in the international market is $1,200 per pound, find the hourly wage rate paid by Firm 1.

(c.) (10%) Suppose the price of coffee in the international market has dropped sharply to $450 per pound, find the total hours of labors Firm 2 will hire.

3. (10%) Currently the most efficient way to grow rice can produce q tons of rice with the long-run total cost function $c(q) = 0.05q^3 - 4q^2 + 200q$. Suppose the rice market is perfectly competitive with market demand $Q = 20,000 - 100P$ when the market price is P per ton. If all rice suppliers are using the same technology, how many suppliers should survive in the long-run equilibrium?

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4. (20%) Suppose a firm's production function is $q = f(L, K) = LK$, where L, K are labor and capital respectively. Assume the wage of labor is 4 (per hour) and the rental rate of capital is 3 (per hour).

(a.) (5%) Find the expansion path.

(b.) (5%) Find the cost function $C(q)$.

(c.) (5%) Does the firm exhibit economies of scale? Explain your answer.

(d.) (5%) In the short run, the capital is fixed at $K = 20$. Does the firm experience diminishing marginal returns to labor? Explain your answer.

5. (20%) A consumer spends all his income on goods A and B and his utility function is

$$u(A, B) = A^{0.2} B^{0.8}.$$

(a.) (10%) Find the price elasticity of demand of the compensated demand function for good A.

(b.) (10%) Assume his income is 120 and the initial prices of goods A and B are $(p_A, p_B) = (2,3)$. Suppose p_A rises to 3. Please find his compensating variation and the change in his consumer surplus.

6. (10%) There are two goods C and D. A consumer's utility function is $u(C, D) = \sqrt{C} + D$.
Are C and D normal goods for the consumer? Explain your answer.