1．$(25 \%)$ A product was sold in two isolated countries， $1 \& 2$ ，with demands $q_{1}=100-2 p$ and $q_{2}=50-$ $p$ respectively．Suppose seller 1 is the monopolized seller in country 1 while seller 2 is the sole provider in country 2．Assume the marginal（average）cost is $\$ 20$ for each seller and there＇s no other cost．
（a）（ $10 \%$ ）If both countries are closed economies so that consumers only buy from sellers in their own countries，find the market price in each country．
（b）$(10 \%)$ If both countries open to trade and two sellers compete in quantity in the international market，find the international market price．（Assume there are no additional transportation costs or tariffs when goods are shipped across countries．）
（c）（5\％）Which country will be the exporter in（b）？How many units are exported？

2．（ $25 \%$ ）Beth has utility function $u\left(x_{1}, x_{2}, x_{3}\right)=x_{1}+x_{2}^{1 / 3} x_{3}^{1 / 3}$ for commodities $1,2 \& 3$ ．Suppose her income is $\$ 180$ and the prices are $\$ 18, \$ 3 \& \$ 2$ for commodities $1,2 \& 3$ separately．
（a）$(5 \%)$ Verify if her preference is homothetic．
（b）$(10 \%)$ Find her quantities demanded for the three commodities．
（c）（ $10 \%$ ）Suppose her income has dropped to $\$ 48$ ，given the same prices，find again her quantities demanded for the three commodities．

3．（ $10 \%$ ）A firm uses only labor（ L ）and capital（ K ）to produce．Suppose the input combinations $x=\left(L_{x}, K_{x}\right)$ and $y=\left(L_{y}, K_{y}\right)$ are on the same isoquant curve convex to the origin as below．Moreover suppose $x$ minimizes the firm＇s cost when input prices are $w$ and $r$ ，while $y$ minimizes the cost for $w$ and $r^{\prime}>r$ ．Let $C(x ; w, r)$ be the firm＇s total cost with input combination $x$ when input prices are $w$ and $r$ ．Similarly for $C\left(y ; w, r^{\prime}\right)$ ．Show that $C\left(y ; w, r^{\prime}\right)>C(y ; w, r)$ ．


## 第2頁，共2頁

4．（ $25 \%$ ）A firm can produce $q=2 L^{1 / 2} K^{1 / 4}$ when using $L$ units of labor and $K$ units of capital．Suppose input prices are $w$ and $r$ for $L$ and $K$ ．
（a）（5\％）Derive the firm＇s short－run total cost function，$C(q ; K)$ ，given any capital level $K$ ．
（b）（ $10 \%$ ）Derive the firm＇s long－run total cost function，$C(q)$ ．
（c）$(10 \%)$ Show that $\frac{c(q ; K)}{q} \geq \frac{c(q)}{q}$ for any $q>0$ and $K$ so that short－run average cost is always higher than its long－run counterpart．

5．（ $15 \%$ ）Consider an exchange economy of two individuals（A and B）and two goods（ 1 and 2）．Suppose utility functions are $u_{A}\left(x_{1}, x_{2}\right)=x_{1}^{1 / 4} x_{2}^{1 / 4}$ and $u_{B}\left(x_{1}, x_{2}\right)=x_{1}^{1 / 2} x_{2}^{1 / 3}$ respectively while endowments are $(100,300)$ for individual A and $(200, K)$ for B．Suppose A and B are price takers for Both commodities． Verify how the equilibrium relative price for the two goods，$p_{1} / p_{2}$ ，will be impacted by $K$ ，B＇s endowment for the second commodity．

